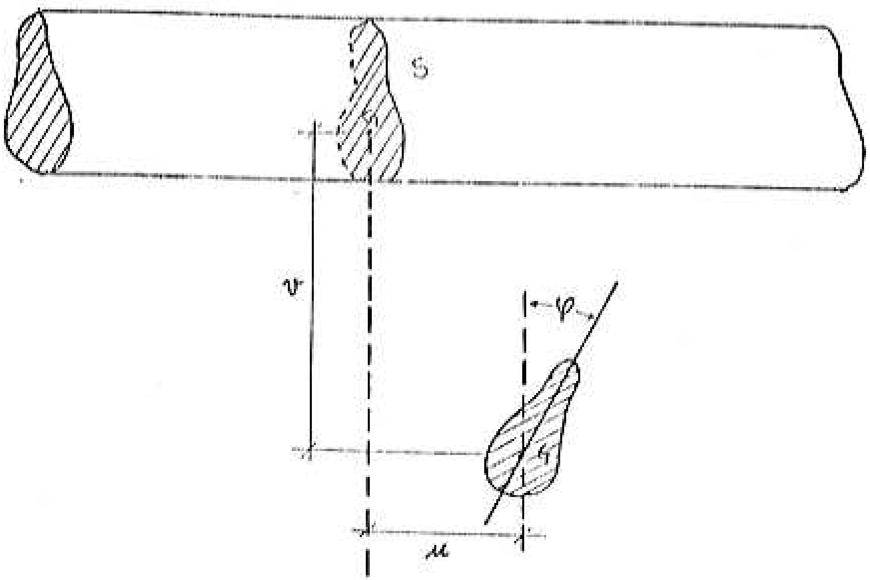
Desplazamientos

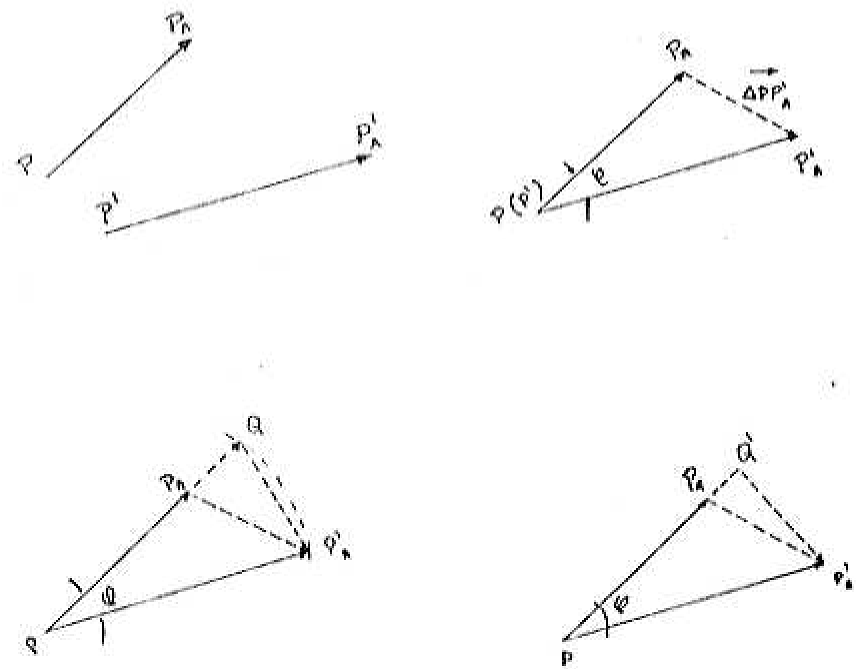
Clase 5

Cálculo por rigidez elástica, barras articuladas, deformación debida a la temperatura, dilatación superficial y volumétrica

DESPLAZAMIENTO DE UNA SECCION

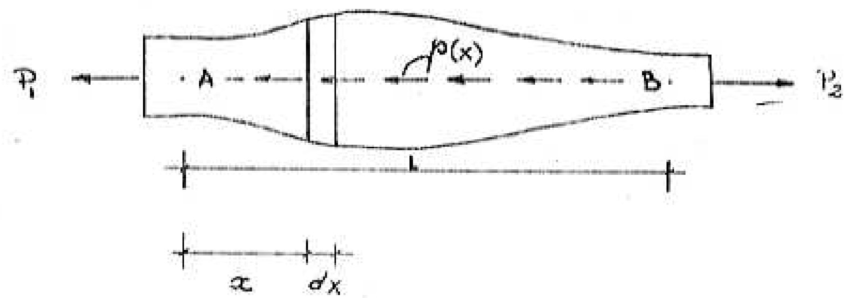


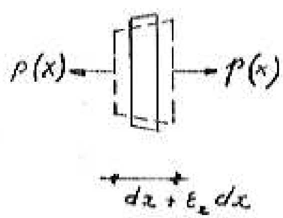
DEFORMACION



$$\varphi = tg\,\varphi = \frac{\overrightarrow{Q'P'_1}}{\overrightarrow{PP_1} + \overrightarrow{P_1Q'}} = \frac{\overrightarrow{Q'P'_1}}{\overrightarrow{PP'_1}}$$

BARRAS CARGADAS AXIALMENTE



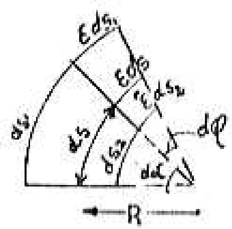


$$\varepsilon_x = \frac{du}{dx}$$

$$U = \int_0^x du + C = \int_0^x \varepsilon_x \, dx + C$$

$$U = \int_0^x \frac{\sigma_x}{E} + C = \int_0^x \frac{P(x)\, dx}{A(x) E} + C$$

P/ BARRAS CURVAS $U = \int_0^{\ell} \frac{P(x)\, ds}{A(x) E}$



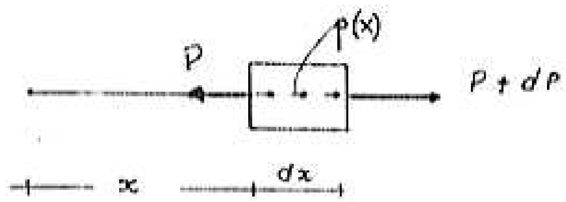
$$d\varphi = \frac{\varepsilon \, ds}{R} = \varepsilon \, d\alpha = \frac{\sigma}{E} d\alpha = \frac{N}{E S} d\alpha$$

a)

$$\frac{du}{dx} = \varepsilon_x = \frac{\sigma_x}{E} = \frac{P(x)}{A(x)\,E}$$

$$P(x) = A(x).E\ \frac{du}{dx}$$

b)



$$\Sigma F_x = 0 \qquad dP + p(x)\,dx = 0$$

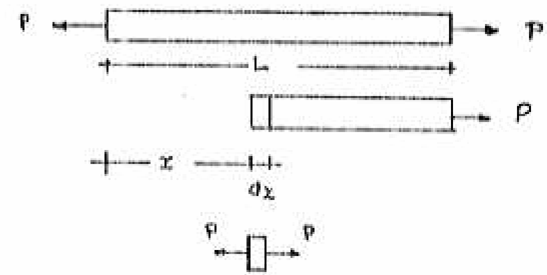
$$\frac{dP}{dx} = -p(x)$$

si $AE = cte$

$$\frac{d}{dx}\left(\frac{du}{dx}\right) = \frac{1}{AE}\ \frac{dP}{dx}$$

$$AE\ \frac{d^2u}{dx^2} = -p(x)$$

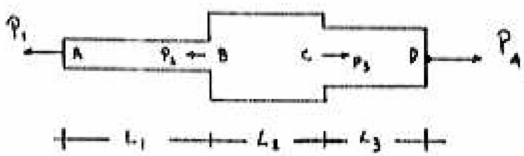
BARRAS DE SECCION CONSTANTE



$$U = \frac{P}{AE}\int_0^x dx + C_1 = \frac{Px}{AE} + C_1 \quad ---- \quad C_1 = 0$$

$$U = \frac{PL}{AE}$$

BARRAS DE SECCION VARIABLE

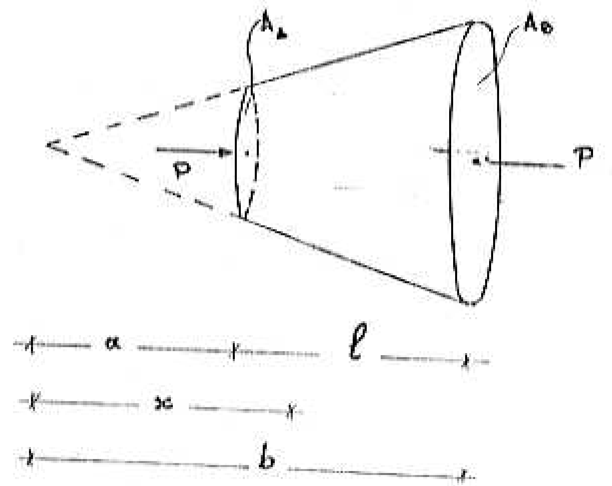


$$U = \int_0^L \frac{P(x)\,dx}{AE} = \int_A^B \frac{N_1\,dx}{AE} + \int_B^C \frac{N_2\,dx}{AE} + \int_C^D \frac{N_3\,dx}{AE}$$

SI P y A SON CTES ENTRE LOS LIMITES

$$U = \sum \frac{N_i L_i}{A_i E}$$

VARIACIÓN CONTINUA DE SECCION

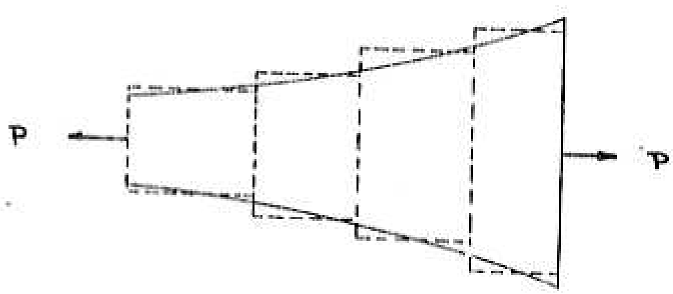


$$A(x) = A_a \frac{x^2}{a^2}$$

$$U = \int_{a}^{a+\ell} \frac{P}{E\,A}\, dx = \frac{P\,a^2}{E\,A_a} \int_{a}^{a+\ell} \frac{1}{x^2}\, dx = \frac{N\,a\,\ell}{E\,A_a\,(a+\ell)}$$

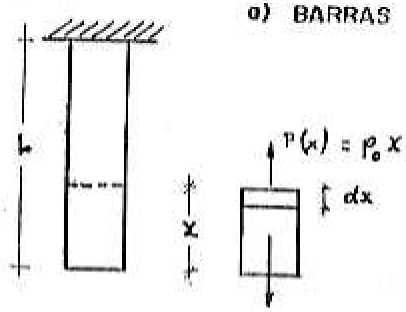
$$\frac{a}{a+\ell} = \sqrt{\frac{A_a}{A_b}}$$

$$U = \frac{P\,\ell}{E\sqrt{A_a A_b}}$$



INFLUENCIA PESO PROPIO

a) BARRAS SECCION CONSTANTE



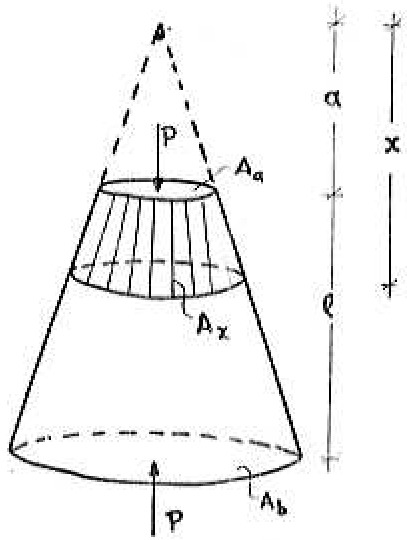
$$P(x) = p_0 x$$

$$U = \int_0^x \frac{P(x)\,dx}{A(x)E} + C_1 = \frac{1}{AE}\int_0^x p_0(x)\,dx + C_1$$

$$U = \frac{-p_0\,(l^2 - x^2)}{2AE}$$

$$U_{MAX} = \frac{WL}{2AE}$$

b) BARRAS DE SECCION VARIABLE



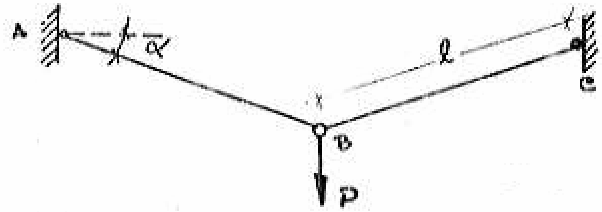
$$A_x = A_a \frac{x^2}{a^2}$$

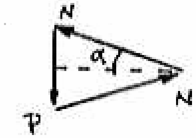
$$P(x) = \frac{\mu}{3}\left(A_x x - A_a a\right) = \frac{\mu A_a}{3}\left(x^3 - a^3\right)$$

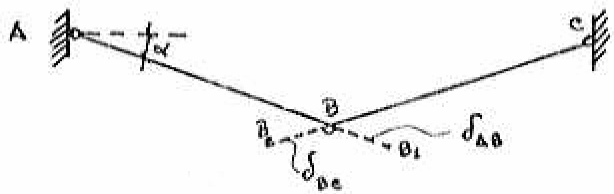
$$U = \int_0^{a+\ell} \frac{\mu}{3E}\left(x^3 - \frac{a^3}{x^2}\right) dx$$

$$U = \frac{\mu \ell^2}{6E}\,\frac{3a+\ell}{a+\ell}$$

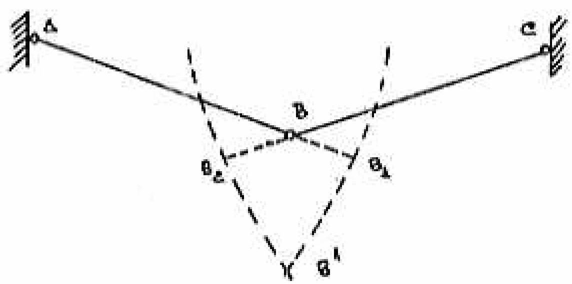
$$U = \frac{\mu \ell^2}{6E}\left(1 + 2\sqrt{\frac{A_a}{A_b}}\right)$$

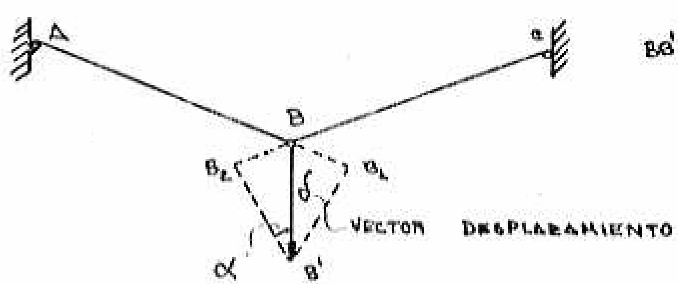




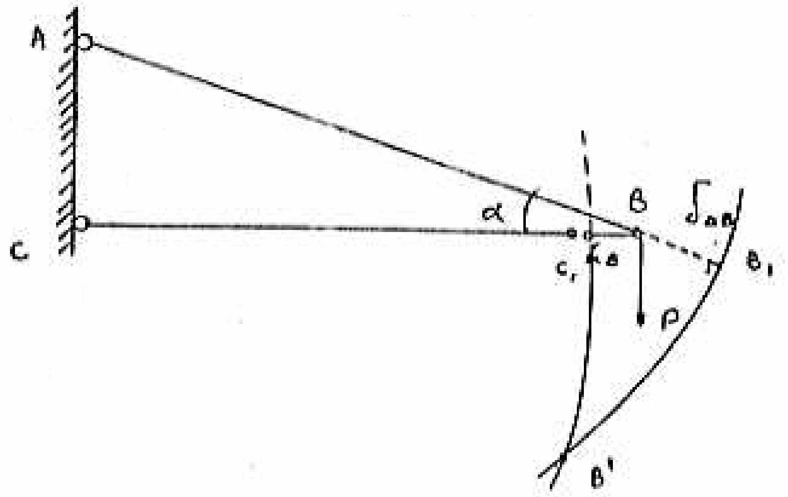


$$\delta_{AB} = \frac{Nl}{AE}$$



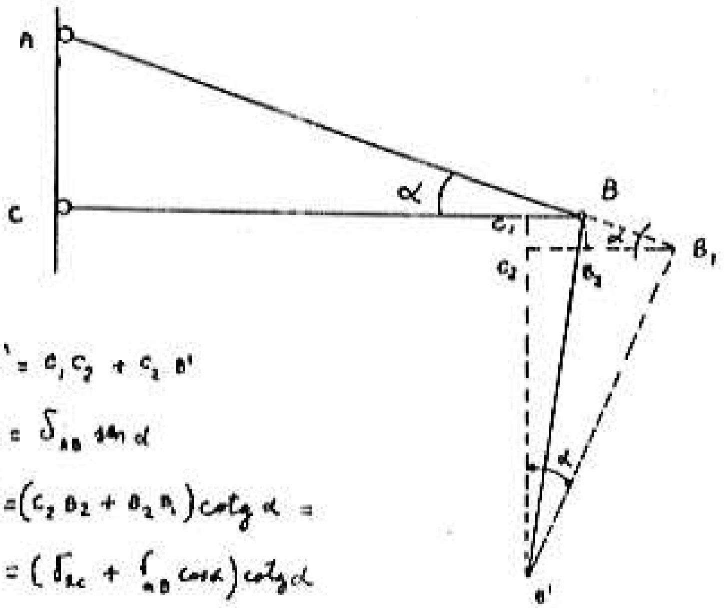


$$BB' = \frac{BB_1}{\operatorname{sen}\alpha} = \frac{Pl}{2AE\operatorname{sen}^2\alpha}$$



δ_{AB} = ALARGAMIENTO AB

δ_{BC} = ACORTAMIENTO AC



C_1B' = DESPLAZAMIENTO VERTICAL

$$C_1B' = C_1C_2 + C_2B'$$

$$C_1C_2 = \delta_{AB} \operatorname{sen} \alpha$$

$$C_2B' = (C_2B_2 + B_2B_1)\operatorname{cotg}\alpha =$$

$$= (\delta_{BC} + \delta_{AB} \cos\alpha)\operatorname{cotg}\alpha$$

$$C_1B' = \delta_{AB} \operatorname{sen}\alpha + (\delta_{BC} + \delta_{AB}\cos\alpha)\operatorname{cotg}\alpha$$

$$C_1B' = \delta_{AB} \operatorname{cosec}\alpha + \delta_{BC} \operatorname{cotg}\alpha$$

DILATACION SUPERFICIAL

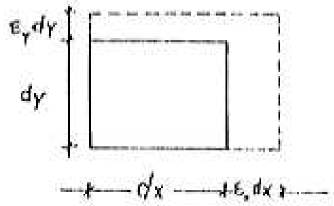
AREA CIRCULO RADIO 1 = π

AREA ELIPSE = $\pi(1+\varepsilon_1)(1+\varepsilon_2) \approx \pi(1+\varepsilon_1+\varepsilon_2)$

$$\varepsilon_S = \frac{\Delta S}{S_0} = \frac{\pi(1+\varepsilon_1+\varepsilon_2)-\pi}{\pi} = \varepsilon_1+\varepsilon_2$$

$$\underline{\varepsilon_S = \varepsilon_1+\varepsilon_2}$$

O SI TOMARAMOS UN ELEMENTO RECTANGULAR :



$$S_0 = dy \cdot dx$$

$$S_f = (dy+\varepsilon_y dy)(dx+\varepsilon_x dx)$$

$$S_f = dy\,dx + \varepsilon_y\,dy\,dx + \cancel{\varepsilon_y\varepsilon_x\,dy\,dx} + \varepsilon_x\,dx\,dy$$

$$\varepsilon_S = \frac{S_f-S_0}{S_0} = \frac{\Delta S}{S_0} = \frac{(dy\,dx+\varepsilon_y\,dy\,dx+\varepsilon_x\,dy\,dx)-dy\,dx}{dy\,dx}$$

$$\underline{\varepsilon_S = \varepsilon_x+\varepsilon_y}$$

$$\varepsilon_x+\varepsilon_y = \varepsilon_1+\varepsilon_2$$

DILATACION VOLUMETRICA

VOLUMEN ESFERA RADIO $1 = \frac{4\pi}{3}$

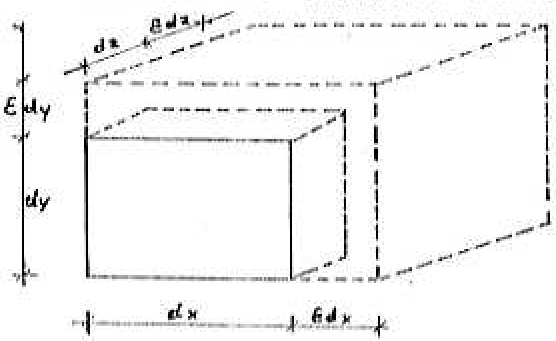
VOLUMEN ELIPSOIDE : $(1+\varepsilon_1)(1+\varepsilon_2)(1+\varepsilon_3)\frac{4\pi}{3} \simeq$

$$\approx \frac{4\pi}{3}\left(1+\varepsilon_1+\varepsilon_2+\varepsilon_3\right)$$

$$\varepsilon_V = \frac{\Delta V}{V} = \frac{\frac{4\pi}{3} + \frac{4\pi}{3}\left(\varepsilon_1+\varepsilon_2+\varepsilon_3\right) - \frac{4\pi}{3}}{\frac{4\pi}{3}} = \varepsilon_1+\varepsilon_2+\varepsilon_3$$

$$\varepsilon_V = \varepsilon_1+\varepsilon_2+\varepsilon_3$$

O TOMANDO UN ELEMENTO PRISMATICO



$$V_0 = dx \cdot dy \cdot dz$$

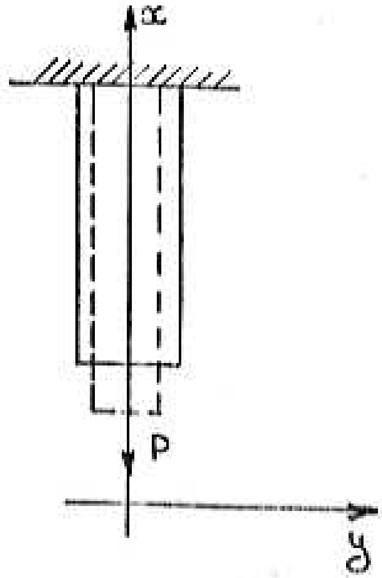
$$V_F = \varepsilon_x(dx+1) \cdot \varepsilon_y(dy+1)\,\varepsilon_z(dz+1)$$

$$V_F = dx \cdot dy \cdot dz\,\left(1+\varepsilon_x+\varepsilon_y+\varepsilon_z\right)$$

$$\varepsilon_V = \frac{V_F - V_0}{V_0} = \frac{\Delta V}{V_0} = \frac{dx\,dy\,dz\,(1+\varepsilon_x+\varepsilon_y+\varepsilon_z) - dx \cdot dy \cdot dz}{dx \cdot dy \cdot dz}$$

$$\varepsilon_V = \varepsilon_x+\varepsilon_y+\varepsilon_z = \varepsilon_1+\varepsilon_2+\varepsilon_3$$

DEFORMACIÓN SUPERFICIAL



$$\varepsilon_y = -\mu\, \varepsilon_x$$

$$\varepsilon_z = -\mu\, \varepsilon_x$$

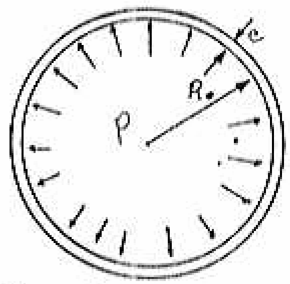
$$\varepsilon_\delta = \varepsilon_y + \varepsilon_z = -2\mu\, \varepsilon_x$$

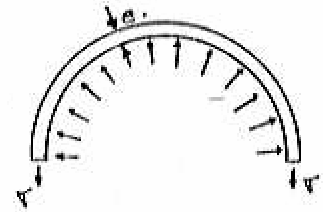
DEFORMACION VOLUMETRICA

$$\varepsilon_y = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$\varepsilon_v = \varepsilon_x - 2\mu\, \varepsilon_y = \varepsilon_x (1 - 2\mu)$$

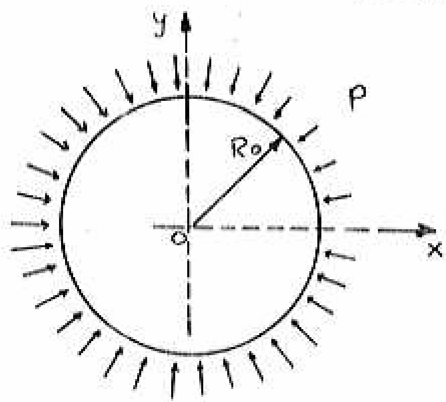
TUBOS Y AROS DE PAREDES DELGADAS





$$\varepsilon_c = \frac{\Delta C}{C_0} = \frac{C - C_0}{C_0} = \frac{2\pi(R_0 + \Delta R) - 2\pi R_0}{2\pi R_0} = \frac{\Delta R}{R_0} = \varepsilon_R$$

RUEDA MACIZA



$$\varepsilon_x = \varepsilon_y = \frac{\sigma_x}{E} - \mu\left(\frac{\sigma_y}{E} - \frac{\sigma_z}{E}\right)$$

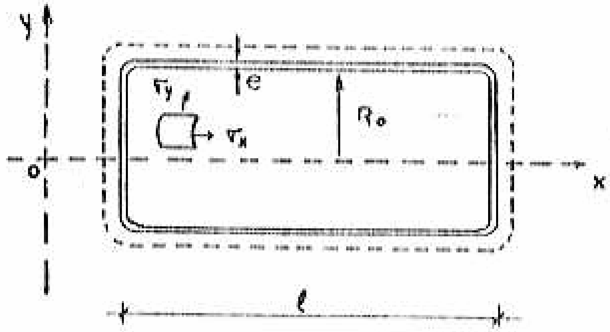
$$\sigma_x = \sigma_y = p$$

$$\sigma_z = 0$$

$$\varepsilon_x = \varepsilon_y = \frac{p}{E}(1-\mu)$$

$$\varepsilon_r = \frac{\Delta R}{R} = \varepsilon_x = \varepsilon_y = \frac{p}{E}(1-\mu)$$

RESERVORIOS CILÍNDRICOS



$$\sigma_y = \frac{P R_o}{e}$$

$$\sigma_x = \frac{P R_o}{2e}$$

$$\sigma_t \approx 0$$

DEFORMACION RADIAL

$$\varepsilon_R = \frac{\sigma_y}{E} - \mu \frac{\sigma_x}{E} = \frac{P R_o}{E e} - \mu \frac{P R_o}{2 E e} = \frac{P R_o}{2 e E} (2 - \mu)$$

DEFORMACION LONGITUDINAL

$$\varepsilon_L = \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} = \frac{P R_o}{2 E e} - \mu \frac{P R_o}{E e} = \frac{P R_o}{2 e E} (1 - 2\mu)$$

DEFORMACION VOLUMETRICA

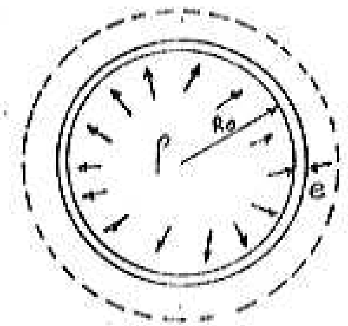
$$\varepsilon_V = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$

$$\varepsilon_1 = \varepsilon_x = \varepsilon_L = \frac{P R_o}{2 e E} (1 - 2\mu)$$

$$\varepsilon_2 = \varepsilon_3 = \varepsilon_y = \frac{P R_o}{2 e E} (2 - \mu)$$

$$\varepsilon_V = \frac{P R_o}{e E} (2{,}5 - 2\mu)$$

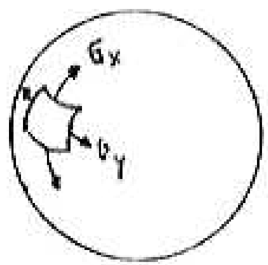
RESERVORIOS ESFÉRICOS



$$\sigma_x = \sigma_y = \frac{pR}{2e} \qquad \sigma_z = 0$$

$$\varepsilon_x = \varepsilon_y = \frac{1}{E}(\sigma_1 - \mu\sigma_2)$$

$$\varepsilon_x = \varepsilon_y = \frac{pR_o}{2Ee}(1-\mu)$$



DEFORMACION RADIAL

$$\varepsilon_x = \varepsilon_y = \varepsilon_R$$

DEFORMACION VOLUMETRICA

$$\varepsilon_v = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 3\varepsilon_1$$

$$\varepsilon_v = 3\varepsilon_x = \frac{3}{2}\,\frac{pR_o}{Ee}(1-\mu)$$

Próxima Clase: Recipientes de paredes delgadas

Fin