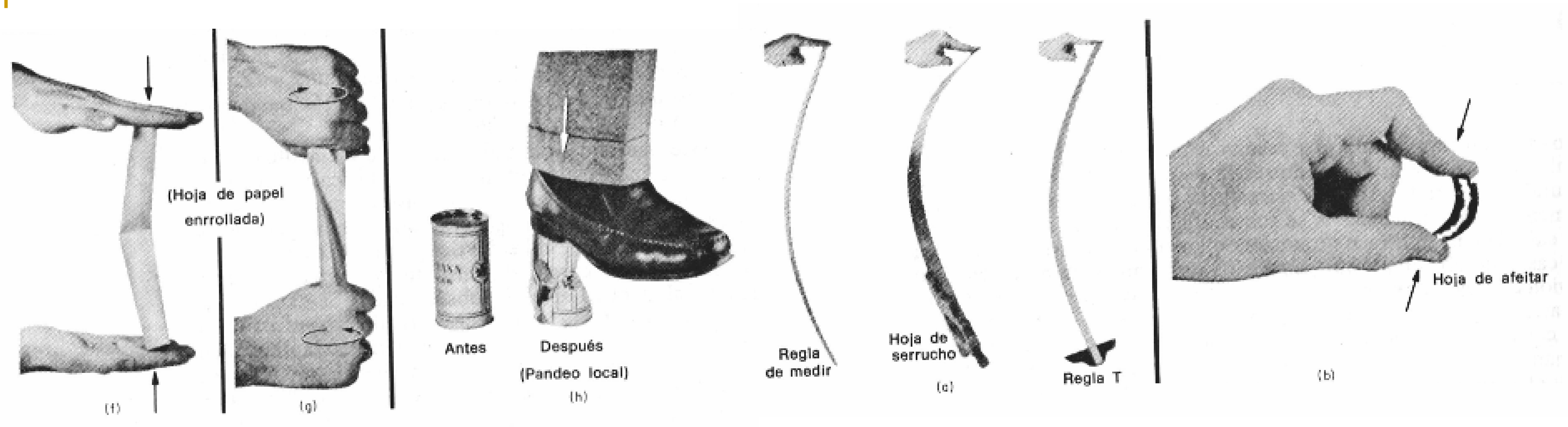
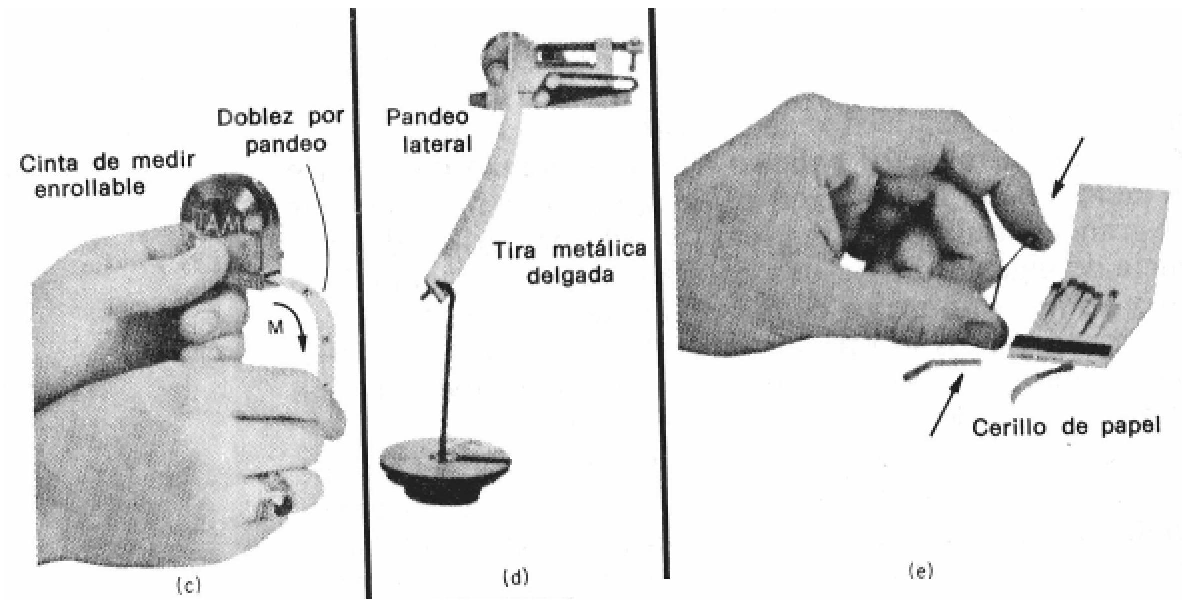
Pandeo

Clase 20

Pandeo Elástico – Pandeo Inelástico

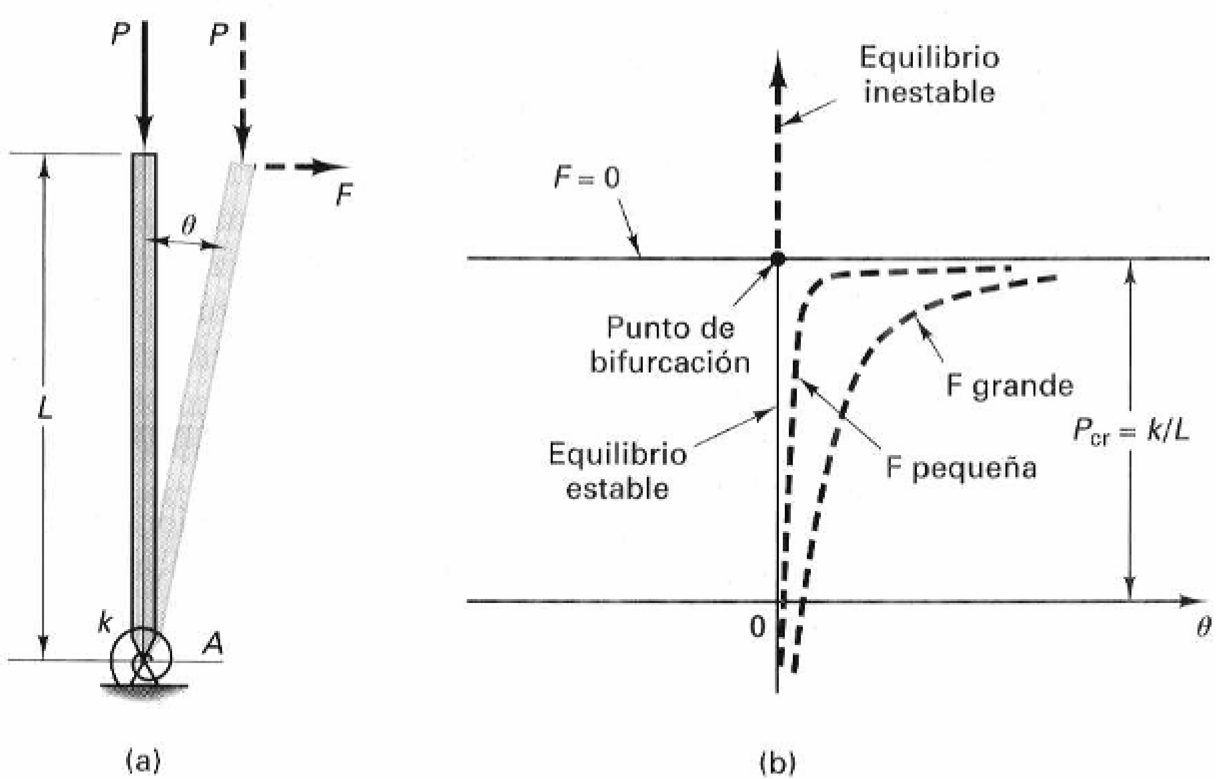

(Hoja de papel
enrrollada)
(f)
(g)
Antes
Después
(Pandeo local)
(h)
Regla
de medir
Hoja de
serrucho
(c)
Regla T
Hoja de afeitar
(b)


Cinta de medir
enrollable
Doblez por
pandeo
M
(c)
Pandeo
lateral
Tira metálica
delgada
(d)
Cerillo de papel
(e)

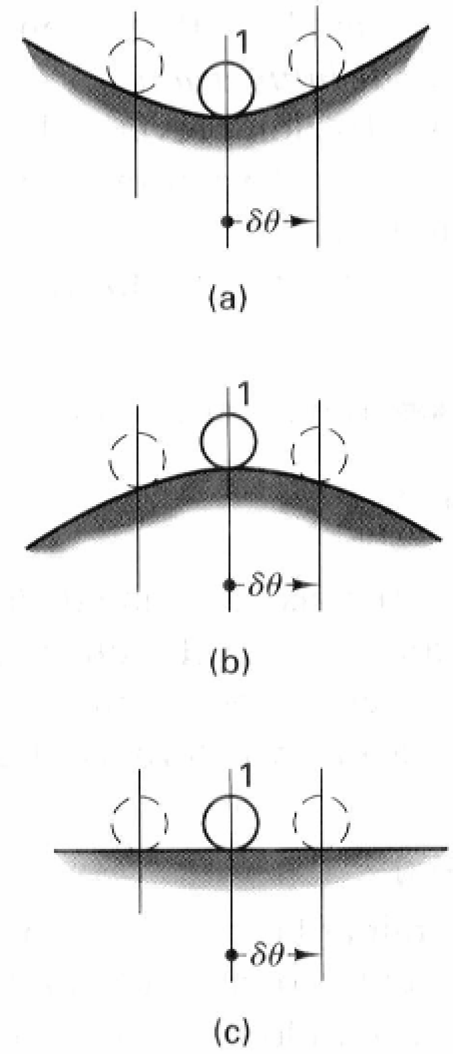
$k\theta > PL\theta$ el sistema es *estable*

y

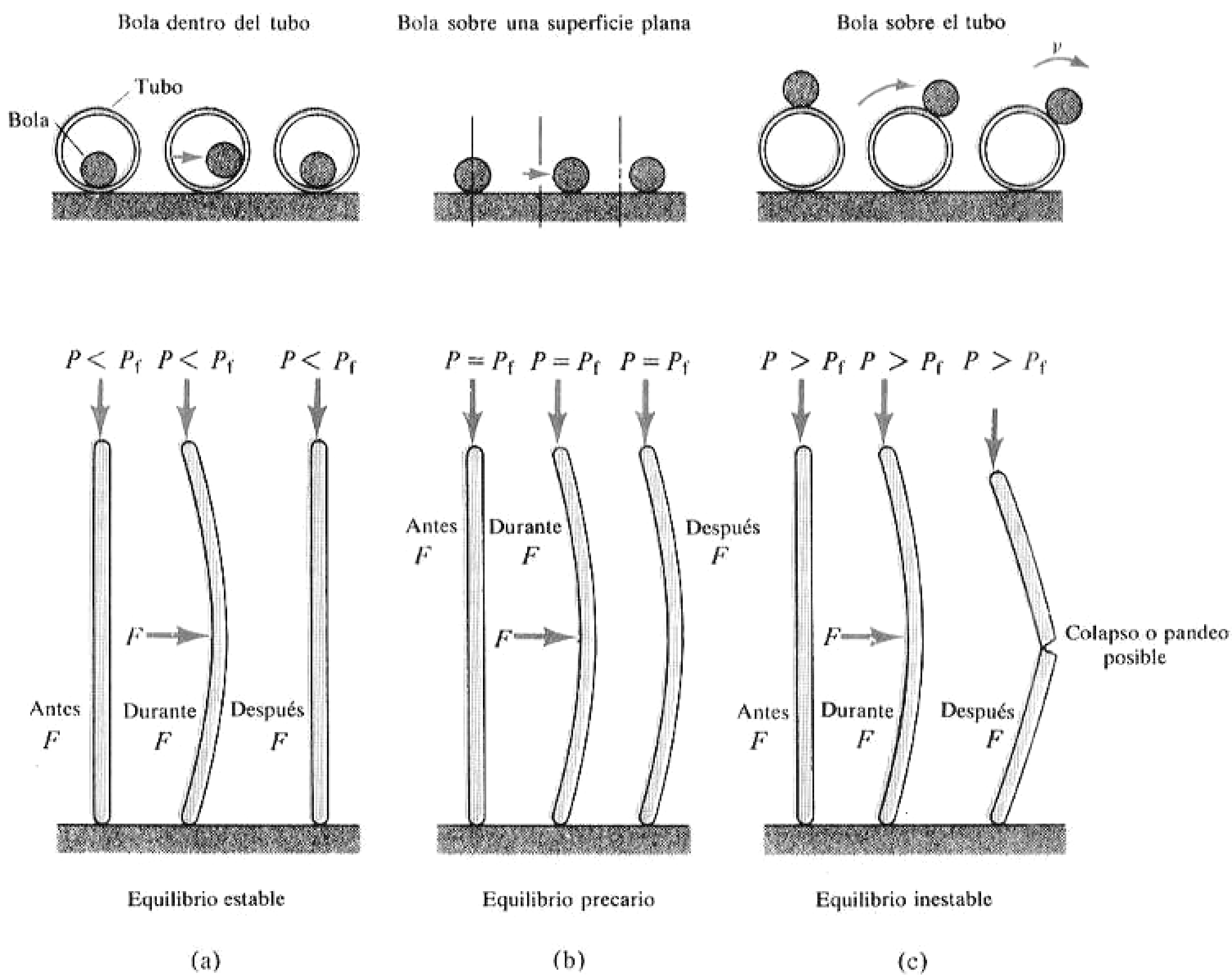
$k\theta < PL\theta$ el sistema es *inestable*



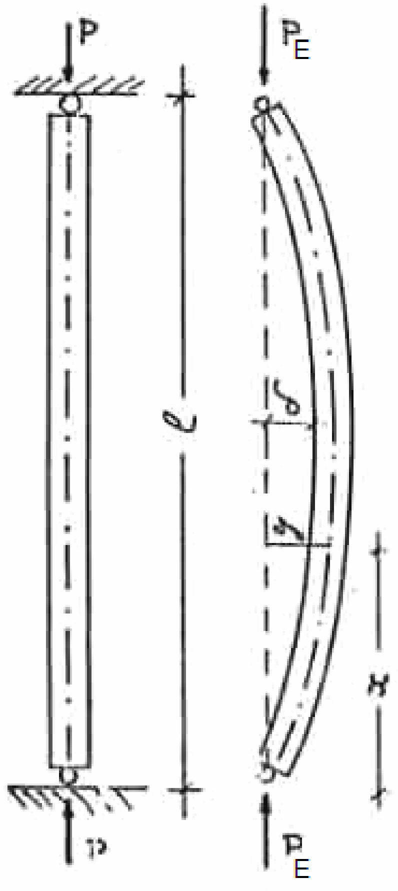
Comportamiento por pandeo de una barra rígida.



Equilibrios (a) estable, (b) inestable y (c) neutro.


Bola dentro del tubo
Bola sobre una superficie plana
Bola sobre el tubo
Tubo
Bola
$P < P_f$
$P = P_f$
$P > P_f$
Antes
Durante
Después
F
Colapso o pandeo posible
Equilibrio estable
Equilibrio precario
Equilibrio inestable
(a)
(b)
(c)

FORMULA DE EULER



$$EI\,\frac{d^2y}{dx^2} = -Py \qquad \text{SI} \qquad k^2 = \frac{P}{EI}$$

$$\frac{d^2y}{dx^2} + k^2y = 0$$

$$y = C_1 \,\text{sen}\, kx + C_2 \cos kx$$

$$y = 0 \quad \text{para} \quad x = 0 \; ; \; x = \ell$$

$$C_2 = 0$$

$$0 = C_1 \,\text{sen}\, k\ell$$

$$\underline{k\ell = n\pi} \qquad n = 0, 1, 2, \ldots$$

PARA $n = 1$

$$\boxed{P_E = P_{crit} = \frac{\pi^2 EI}{\ell^2}}$$

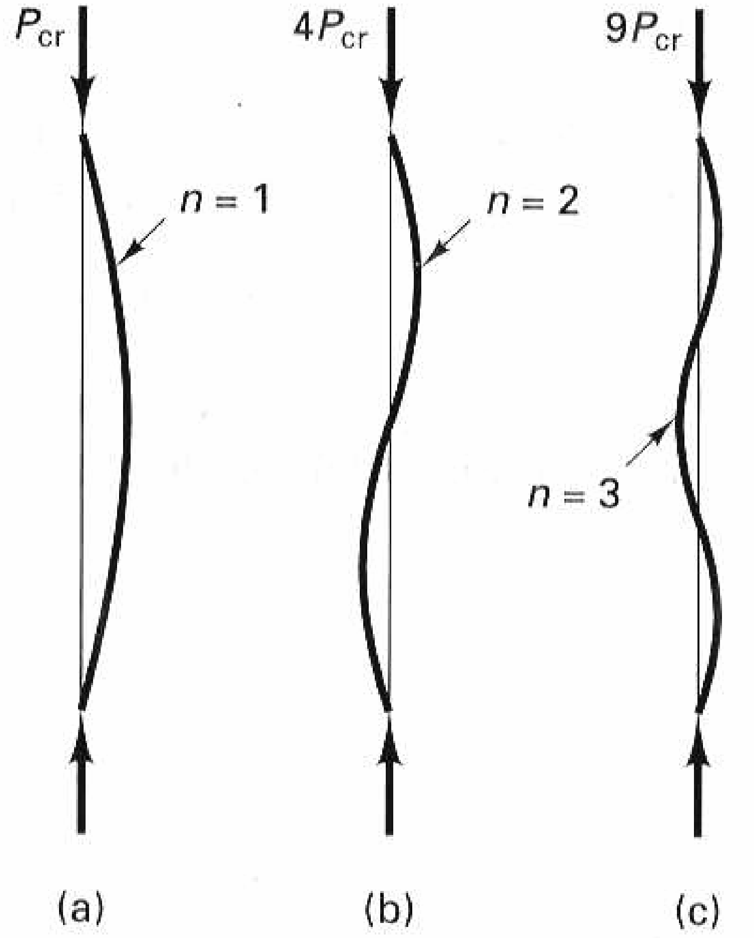
CASOS TEÓRICOS

para $x = \frac{\ell}{2}$ $y = \delta$

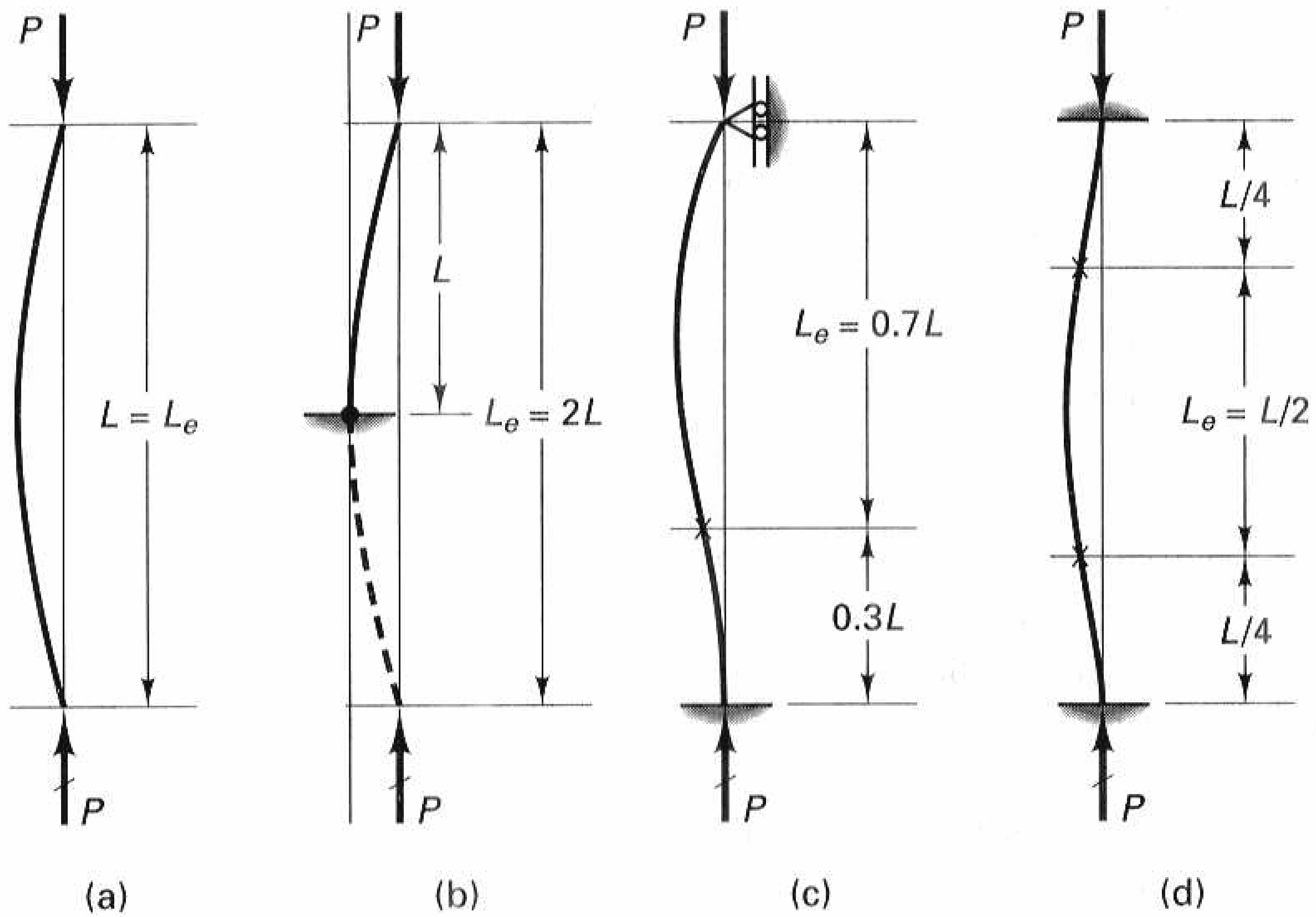
$$\delta = C_1 \operatorname{sen} \frac{n\pi}{\ell} \frac{\ell}{2}$$

$$\delta = C_1$$

$$\boxed{y = \delta \operatorname{sen} \frac{n\pi}{\ell} x}$$



Tres primeros modos de pandeo para una columna articulada en ambos extremos.



Longitudes efectivas de columnas con diferentes restricciones.

ESBELTEZ GEOMETRICA: $\dfrac{\ell}{a}$ a = menor dimension

ESBELTEZ MECANICA: $\lambda = \dfrac{\ell_p}{i_{min.}}$ i = radio de giro minimo

CURVA DE EULER

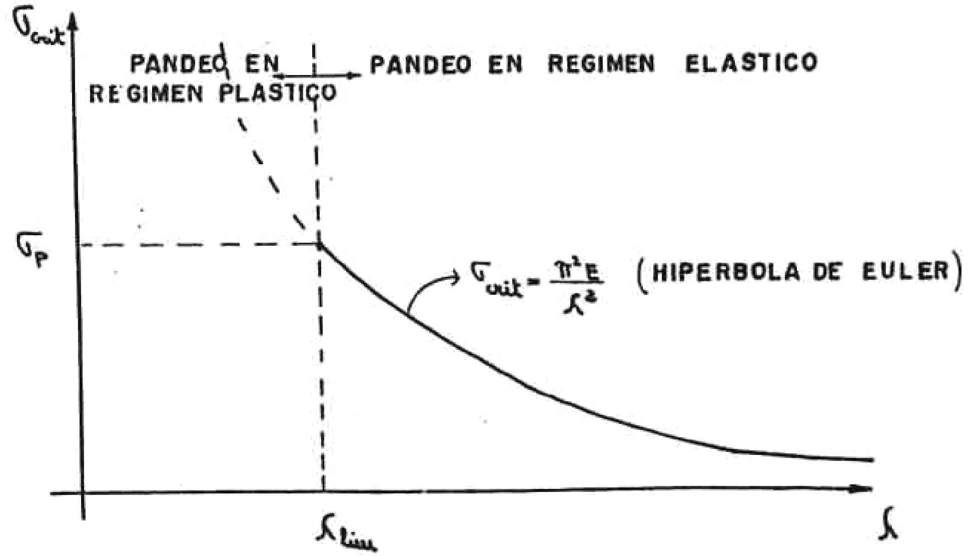
$$P_{crit} = \frac{\pi^2 E I}{\ell_p^2} = \pi^2 E A \left(\frac{i}{\ell_p}\right)^2 = \pi^2 \frac{EA}{\lambda^2}$$

$$\boxed{\sigma_{crit} = \frac{\pi^2 E}{\lambda^2}}$$

$$\sigma_{crit} = \frac{\pi^2 E}{\lambda^2}$$

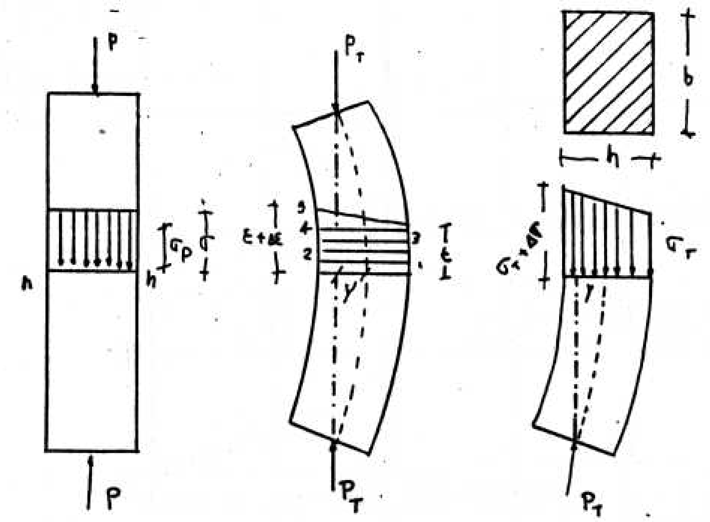
$$\sigma_p = \frac{\pi^2 E}{\lambda_{lim}^2}$$

$$\boxed{\lambda_{lim} = \pi \sqrt{\frac{E}{\sigma_p}}}$$

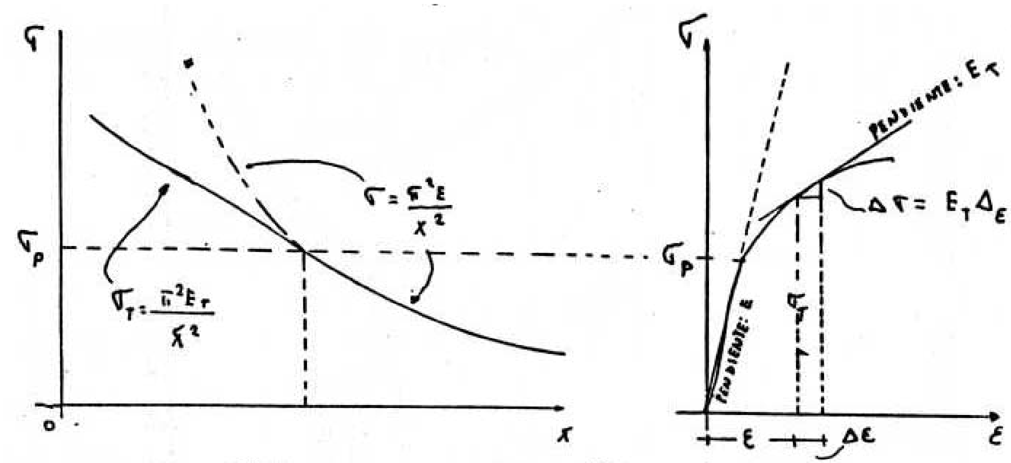


PANDEO INELÁSTICO

FORMULA DEL MODULO TANGENCIAL

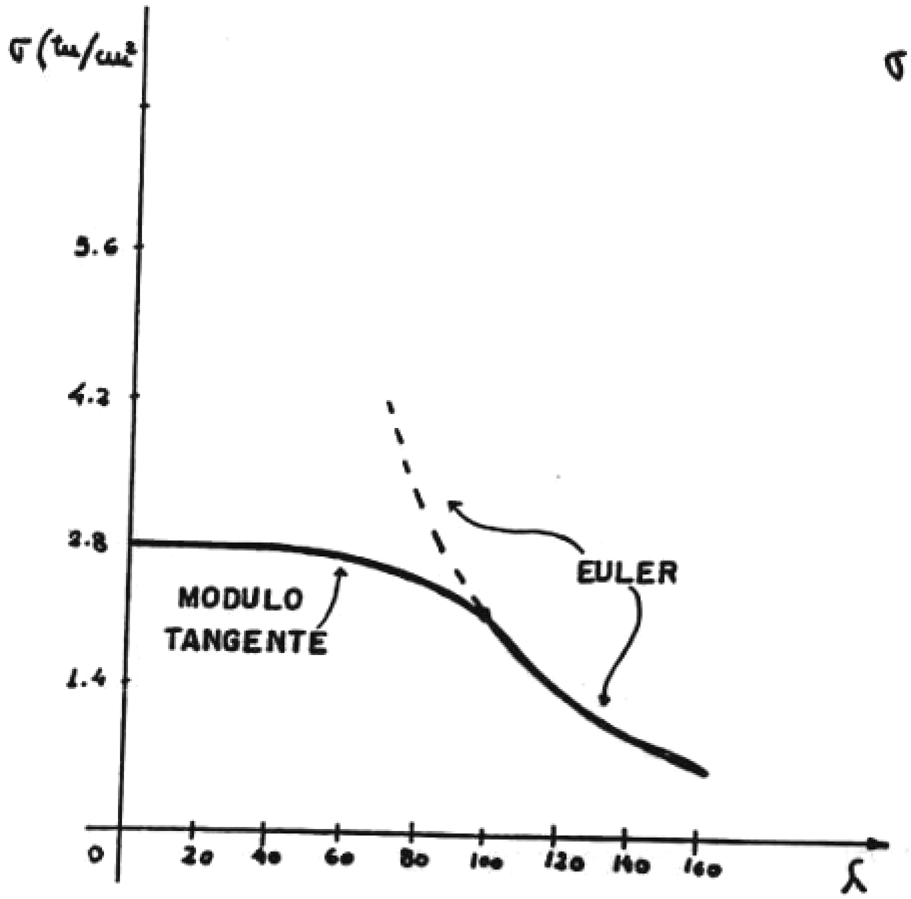


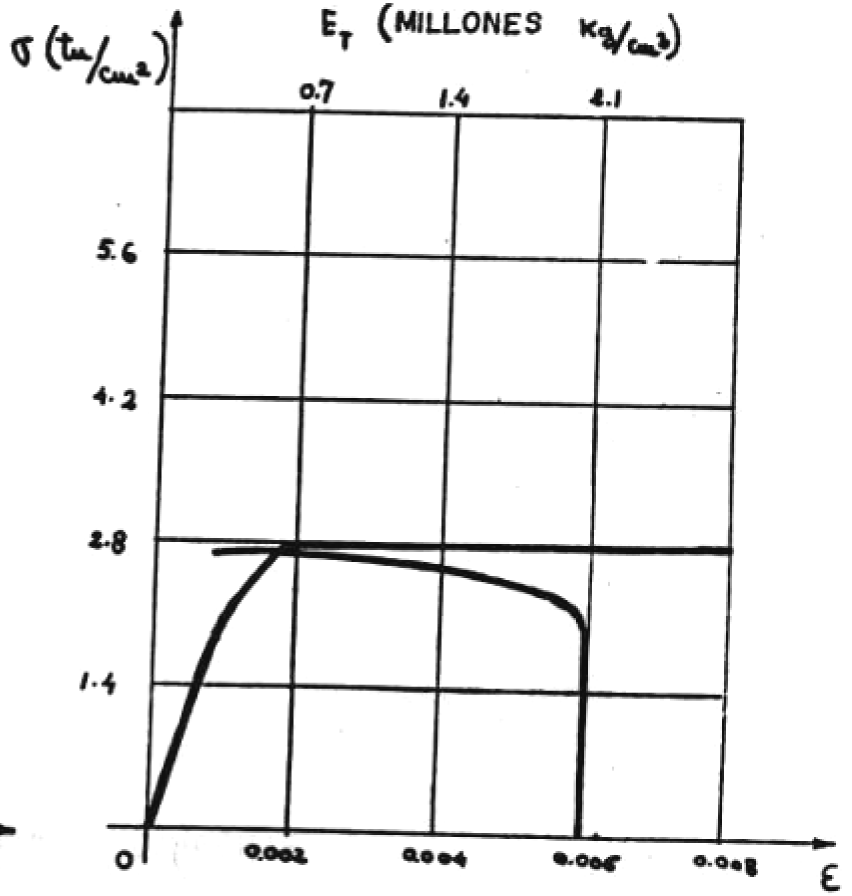
$$P_T \cdot y = \frac{\frac{\Delta\sigma}{2} \cdot I}{\frac{h}{2}} \quad ; \quad P_T \cdot y = \frac{E_T \Delta_\varepsilon I}{h} \quad ; \quad P_T \cdot y = \frac{E_T I}{R} \quad ; \quad E_T I \frac{d^2 y}{dx^2} = -P_T y$$



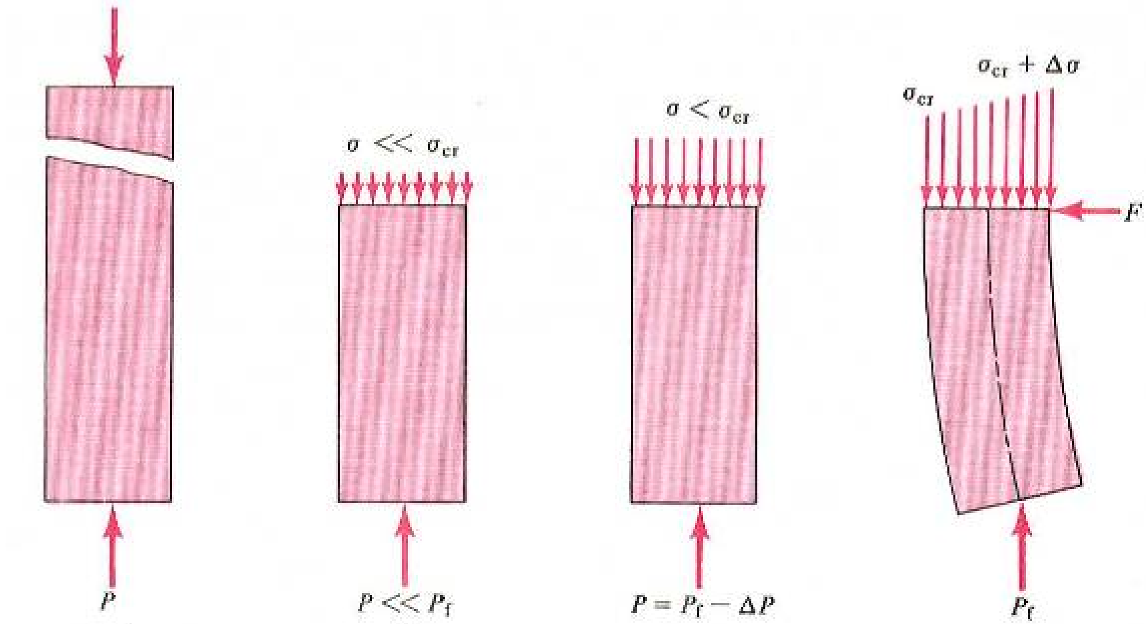
$$P_T = \frac{\pi^2 E_T I}{\ell_p^2} \quad ; \quad \sigma_T = \frac{\pi^2 E_T}{\lambda^2}$$

PANDEO INELÁSTICO – MODULO TANGENTE

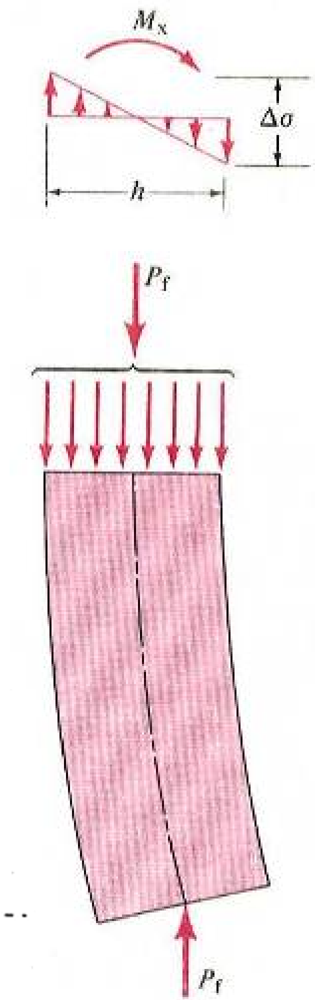


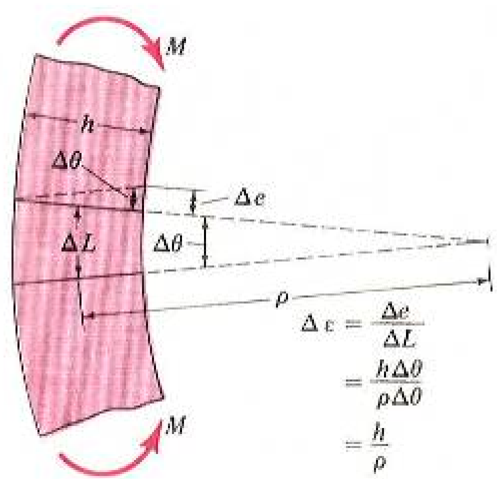


PANDEO INELÁSTICO – FORMULA DEL DOBLE MÓDULO TANGENCIAL

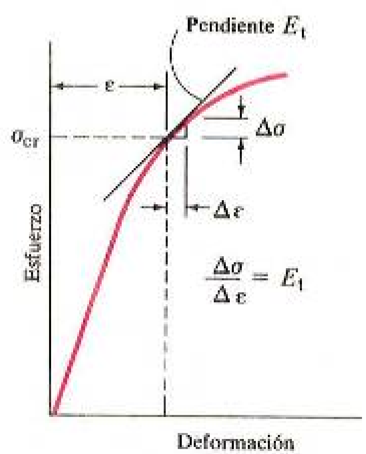


(a) (b) (c) (d)

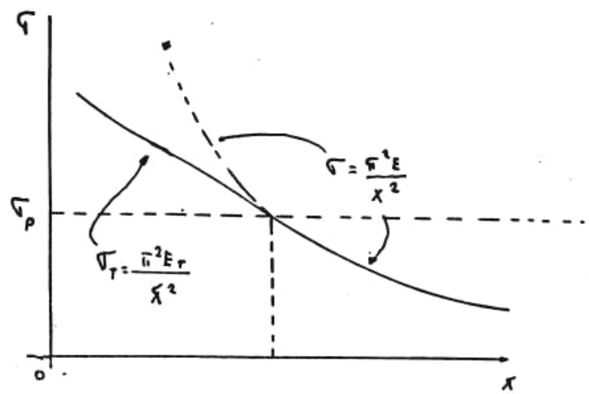




(e)

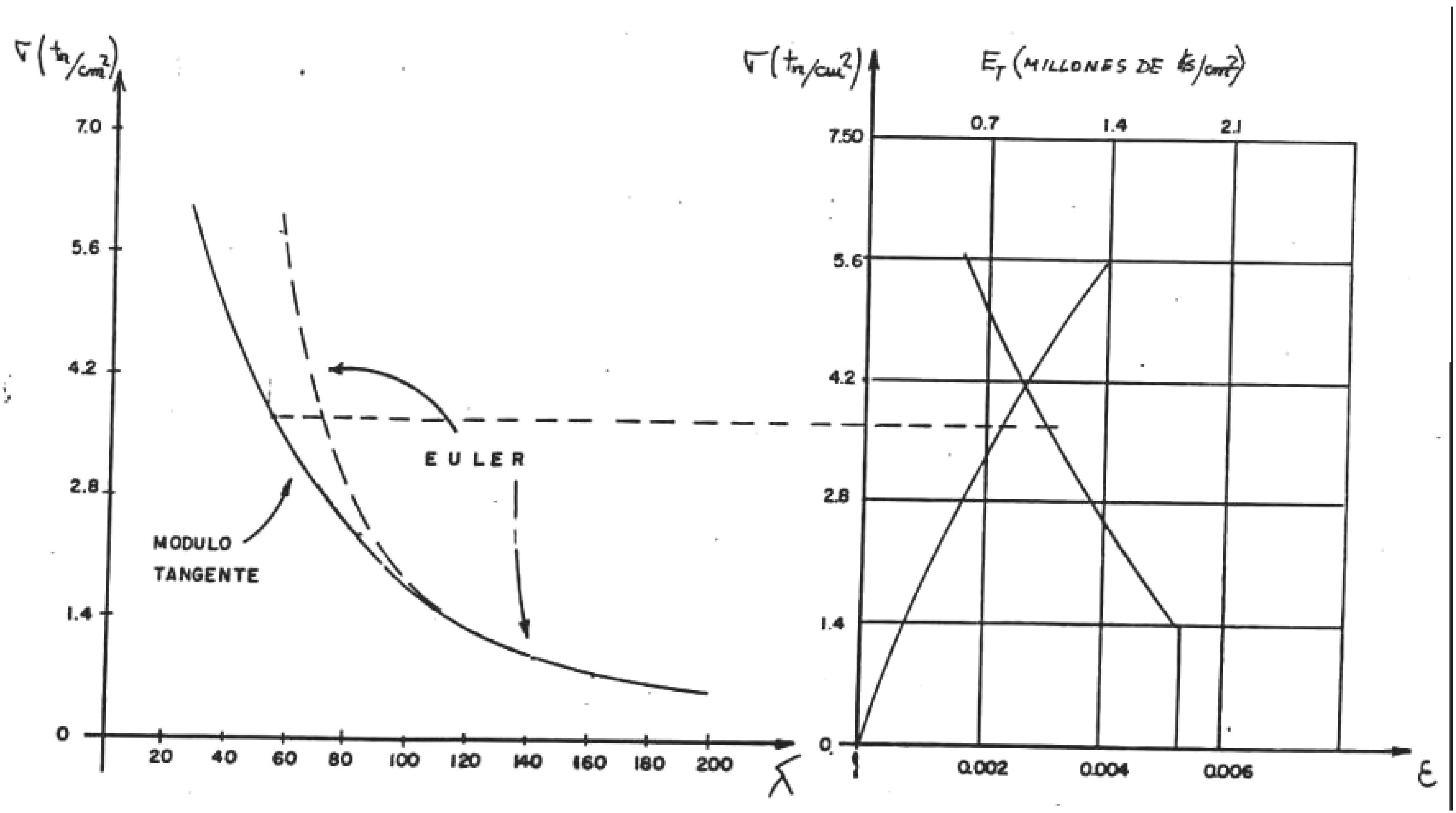


(f)



$P_T = \dfrac{\pi^2 E_T I}{\ell_p^2}$; $\sigma_T = \dfrac{\pi^2 E_T}{\lambda^2}$

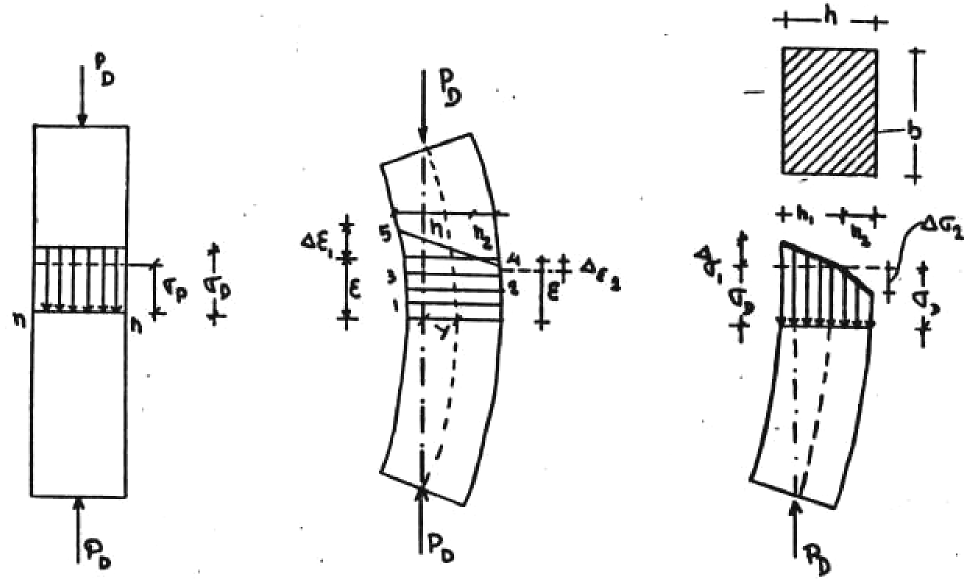
PANDEO INELASTICO – MODULO TANGENTE



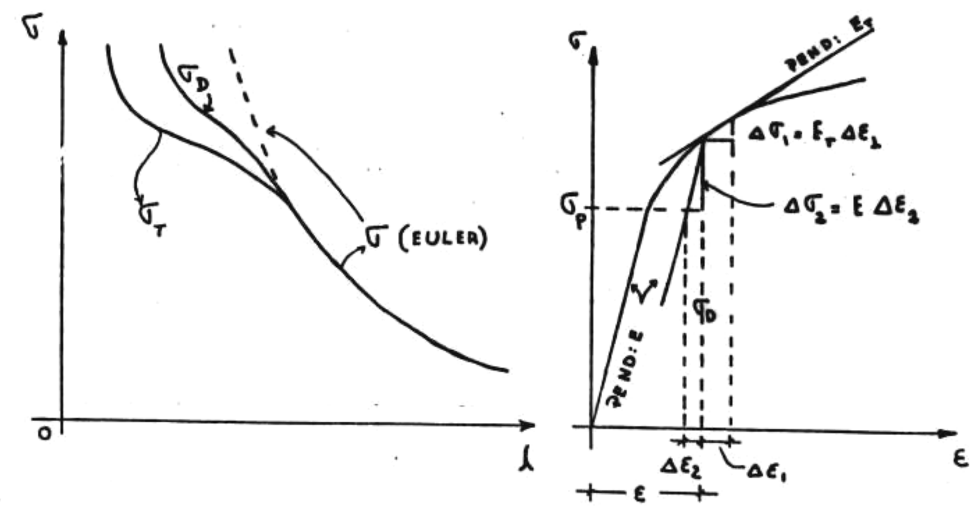
Acero inoxidable

PANDEO INELASTICO:

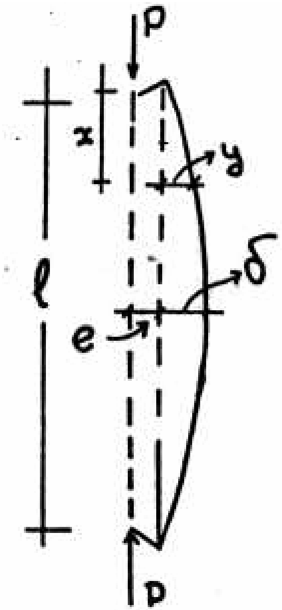
TEORÌA DEL DOBLE MÓDULO



$$P_D = \frac{\pi^2 E_D I}{\ell_p^2} \qquad \sigma_D = \frac{\pi^2 E_D}{\lambda^2} \qquad E_D = \frac{4\,E_T}{\left(1+\sqrt{\frac{E_T}{E}}\right)^2}$$



FORMULA DE LA SECANTE O DE SCHEFFLER

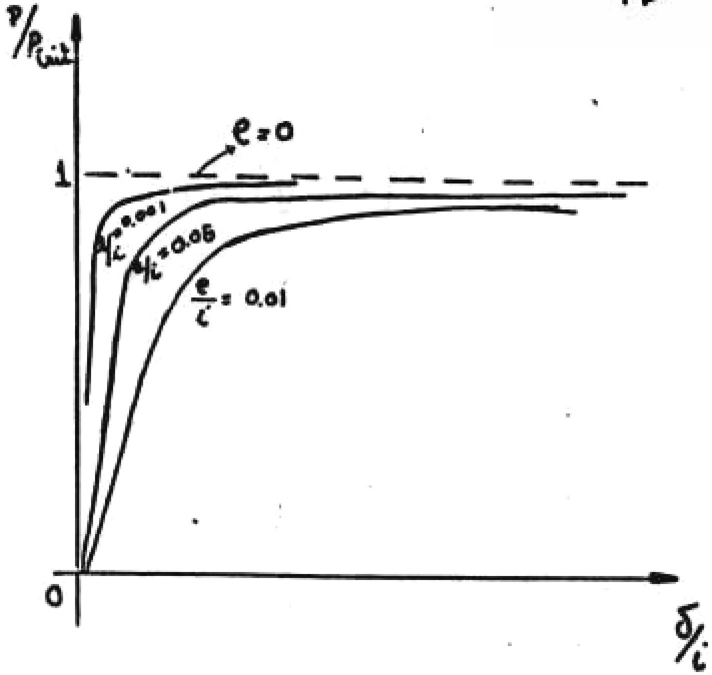


$$M = P(e+y) \qquad K^2 = \frac{P}{EI}$$

$$\frac{d^2y}{dx^2} + K^2 y = -K^2 e$$

$$y = e\left(tg\,\frac{k\ell}{2}\,sen\,kx + cos\,kx - 1\right)$$

$$\delta = e\left(sec\,\frac{k\ell}{2} - 1\right)$$



$$M_{max} = Pe\,sec\,\frac{k\ell}{2}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}\,c}{I}$$

$$i = \sqrt{\frac{I}{A}}$$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{e.c}{i^2}\,sec\left(\frac{k\ell}{2}\right)\right]$$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{e.c}{i^2}\,sec\left(\frac{\ell}{2i}\sqrt{\frac{P}{AE}}\right)\right]$$

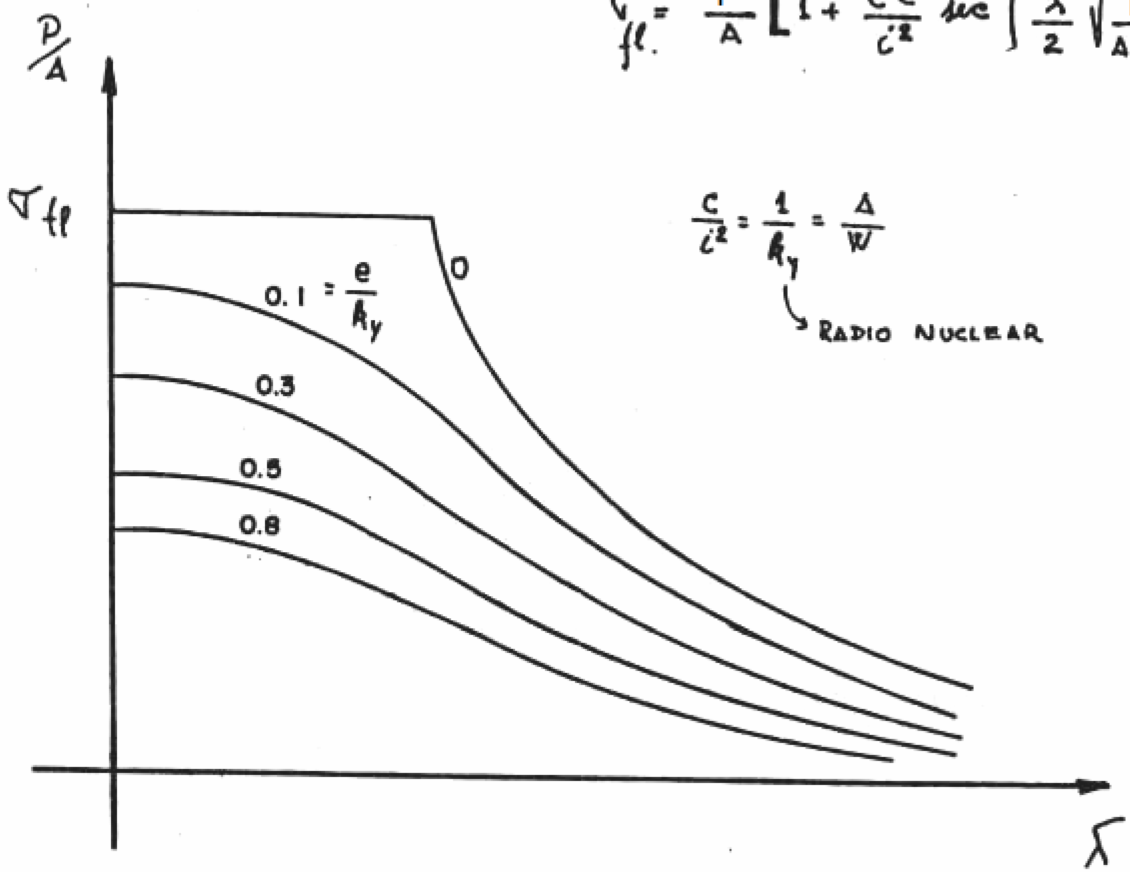
$$\frac{P_{crit}}{A} = \frac{\sigma_{f\ell}}{1 + \frac{e\,c}{i^2}\,sec\left(\frac{\ell}{2i}\sqrt{\frac{P}{AE}}\right)}$$

$$\frac{nP}{A} = \frac{\sigma_{f\ell}}{1 + \frac{e\,c}{i^2}\,sec\left(\frac{\ell}{2i}\sqrt{\frac{nP}{AE}}\right)}$$

$$\sigma_{fl} = \frac{P}{A}\left[1 + \frac{ec}{i^2}\sec\left(\frac{\lambda}{2}\sqrt{\frac{P}{AE}}\right)\right]$$

$$\frac{c}{i^2} = \frac{1}{k_y} = \frac{A}{W}$$

RADIO NUCLEAR



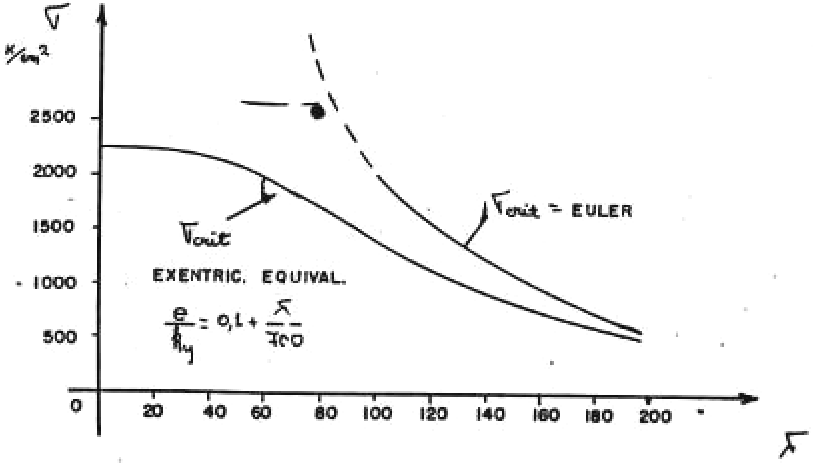
$$\frac{\sigma_{fl}}{n} = \frac{P}{A} + \frac{Pe}{W}\sec\left(\frac{\lambda}{2}\sqrt{\frac{nP}{\Delta E}}\right)$$

FORMULA PARA DIMENSIONAMIENTO

Pcritico depende

- IMPERFECCIONES
- RESISTENCIA
- ESTABILIDAD

EXCENTRICIDAD EQUIVALENTE



$\frac{ec}{i^2}$ $\frac{e}{k_y} = 0.25$ P/ COLUMNAS ARTICULADAS EN LOS EXTREMOS

$\frac{e}{\ell} = \frac{1}{400}$; (KAYSER) RECOMENDABLE P/ TIMOSHENKO

$\frac{e}{k_y} = 0.06 ; 0.07$ (MARSTON; DENSEN)

$\frac{e}{k_y} = 0.15 \rightarrow 0.6$ (MONORIEF)

$\frac{e}{\ell} = 0.001$ (SALMON)

$\frac{e}{k_y} = 0.1 + \frac{\lambda}{700}$ (PRICHARD)

$\frac{e}{k_y} = 0.1 + \frac{\lambda}{1000}$ (BASQUIN)

$e = \frac{1}{20} + \frac{\ell}{500}$ (DIN 4114)

P/ PERFILES DE ALMA LLENA

$e = \frac{\ell}{750} + \frac{h}{40}$

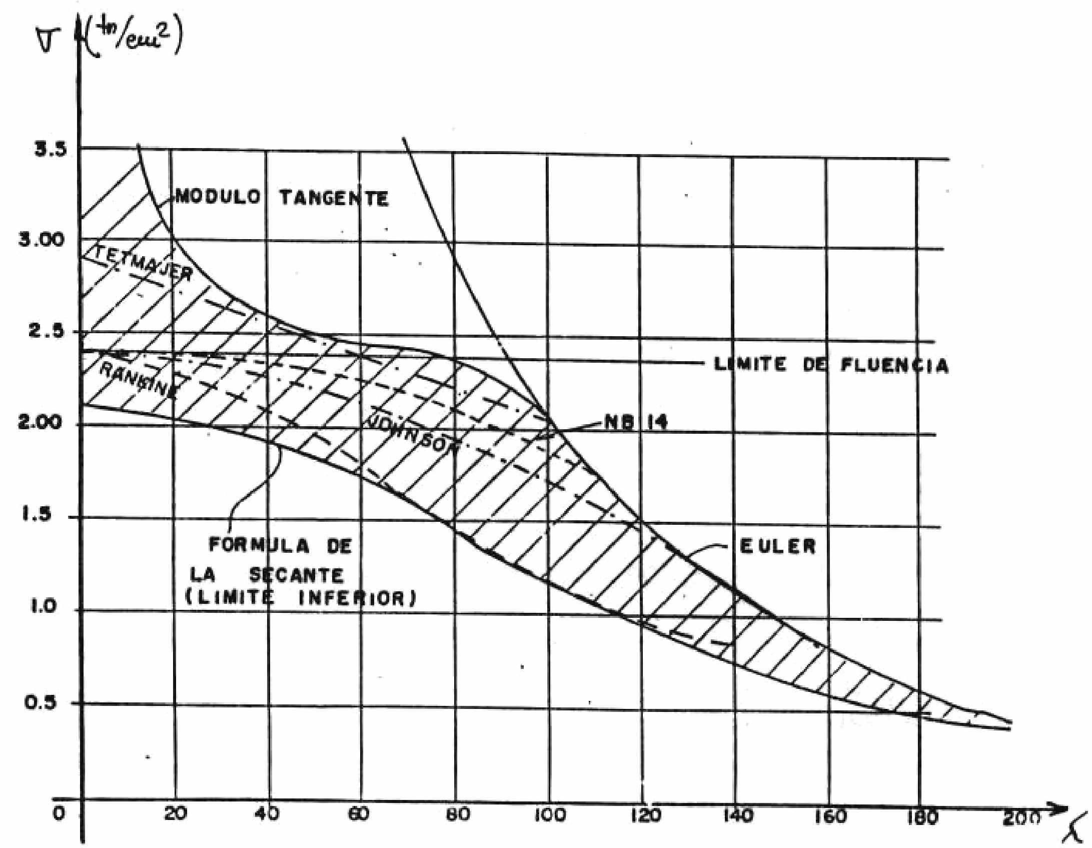
P/PERFILES COMPUESTOS

$e = \frac{\ell}{750} + \frac{h}{40} + \frac{h}{160}$

h = DIMENSION DE LA SECCION EN EL PLANO DEL PANDEO

FÓRMULAS EMPÍRICAS

- LINEA RECTA : $\sigma_{crit} = \sigma_0 - \alpha\lambda$
- GORDON RANKINE : $\sigma_{crit} = \dfrac{\sigma_0}{1 + \beta\lambda^2}$
- PARABOLICA (JOHNSON) : $\sigma_{crit} = \sigma_0 - c\lambda^2$
- TETMAJER : $\sigma_{crit} = \sigma_0 - a\lambda + b\lambda^2$



ACERO DULCE : $\sigma_f = 2.4\ tn/cm^2$

$\sigma_p = 1{,}9\ tn/cm^2$

FORMULAS USUALES

CHICAGO BUILDING CODE — P/ ACERO

$30 < \lambda < 120$ $\quad \bar{\sigma}_{crit} = 1125 - 4{,}921\,\lambda$ $\quad$ LINEA RECTA

$\lambda < 30$ $\quad \bar{\sigma}_{crit} = 984\ K/cm^2$

$\lambda > 120$ $\quad \bar{\sigma}_{crit} = \dfrac{\pi^2 E}{\lambda^2 \eta}$ $\quad \eta = 2{,}7$ $\quad$ EULER

COLUMN RESEARCH COUNCIL

$\lambda < \lambda_{lim}$

$$\frac{\sigma_{crit}}{\sigma_f} = 1 - \frac{\lambda^2}{2\lambda^2_{lim}}$$

$$\frac{\bar{\sigma}}{\sigma_f} = \frac{1 - \dfrac{\lambda^2}{2\lambda^2_{lim}}}{n_1} \qquad n_1 = \frac{5}{3} + \frac{3\lambda}{8\lambda_{lim}} - \frac{\lambda^3}{8\lambda^3_{lim}}$$

$\lambda > \lambda_{lim}$

$$\frac{\sigma_{crit}}{\sigma_f} = \frac{\lambda^2_{lim}}{2\lambda^2}$$

$$\frac{\bar{\sigma}}{\sigma_f} = \frac{\lambda^2_{lim}}{2 n_2 \lambda^2} \qquad n_2 = \frac{23}{12} \approx 1{,}92$$

AMERICAN INSTITUTE OF STEEL CONSTRUCTION: (AISC)

$\lambda < 120$ $\quad \bar{\sigma}_{crit} = 1200 - 0{,}0341\,\lambda^2$ $\quad$ (PARABOLICA)

$\lambda > 120$ $\quad \bar{\sigma}_{crit} = \dfrac{1265}{1 + \dfrac{\lambda^2}{18.000}}$ $\quad$ (GORDON RANKINE)

NEW YORK BUILDING CODE P/ ACERO ESTRUCTURAL

$\lambda < 60$ $\quad \bar{\sigma} = 1055$

$\lambda > 60$ $\quad \bar{\sigma}_{crit} = \dfrac{1265}{1 + \dfrac{\lambda^2}{18.000}}$ (GORDON RANKINE)

NB – 14 P/ACERO ESTRUCTURAL A36

$\lambda = 0$ $\quad \bar{\sigma} = 1200\ ^{k}/_{cm^2}$

$0 < \lambda < 105$ $\quad \bar{\sigma}_{crit} = 1200 - 0{,}023\,\lambda^2$ PARABOLICA

$\lambda > 105$ $\quad \bar{\sigma}_{crit} = \dfrac{10.363.000}{\lambda^2}$ (EULER $- \ \eta = 2$)

P/ ACERO $\sigma_{fl} = 3500\ ^{K}/_{cm^2}$

$\lambda = 0$ $\quad \bar{\sigma} = 1750\ ^{K}/_{cm^2}$

$0 < \lambda < 86$ $\quad \bar{\sigma}_{crit} = 1750 - 0{,}0473\,\lambda^2$ PARABOLICA

$\lambda > 86$ $\quad \bar{\sigma}_{crit} = \dfrac{10.363.000}{\lambda^2}$ EULER

NB – 11 MADERA

$\lambda < 40$ $\quad \bar{\sigma}_{cr} = \bar{\sigma}_c$

$40 < \lambda < \lambda_{lim}$ $\quad \bar{\sigma}_{crit} = \bar{\sigma}_c \left(1 - \dfrac{1}{3}\,\dfrac{\lambda - 40}{\lambda_{lim} - 40}\right)$

$\lambda > \lambda_{lim}$ $\quad \bar{\sigma}_{crit} = \dfrac{\pi^2 E}{4\lambda^2} = \dfrac{2}{3}\bar{\sigma}_c \left(\dfrac{\lambda_{lim}}{\lambda}\right)^2$ EULER ; $\eta = 4$

Final del Programa de Mecánica de Materiales I

Fin