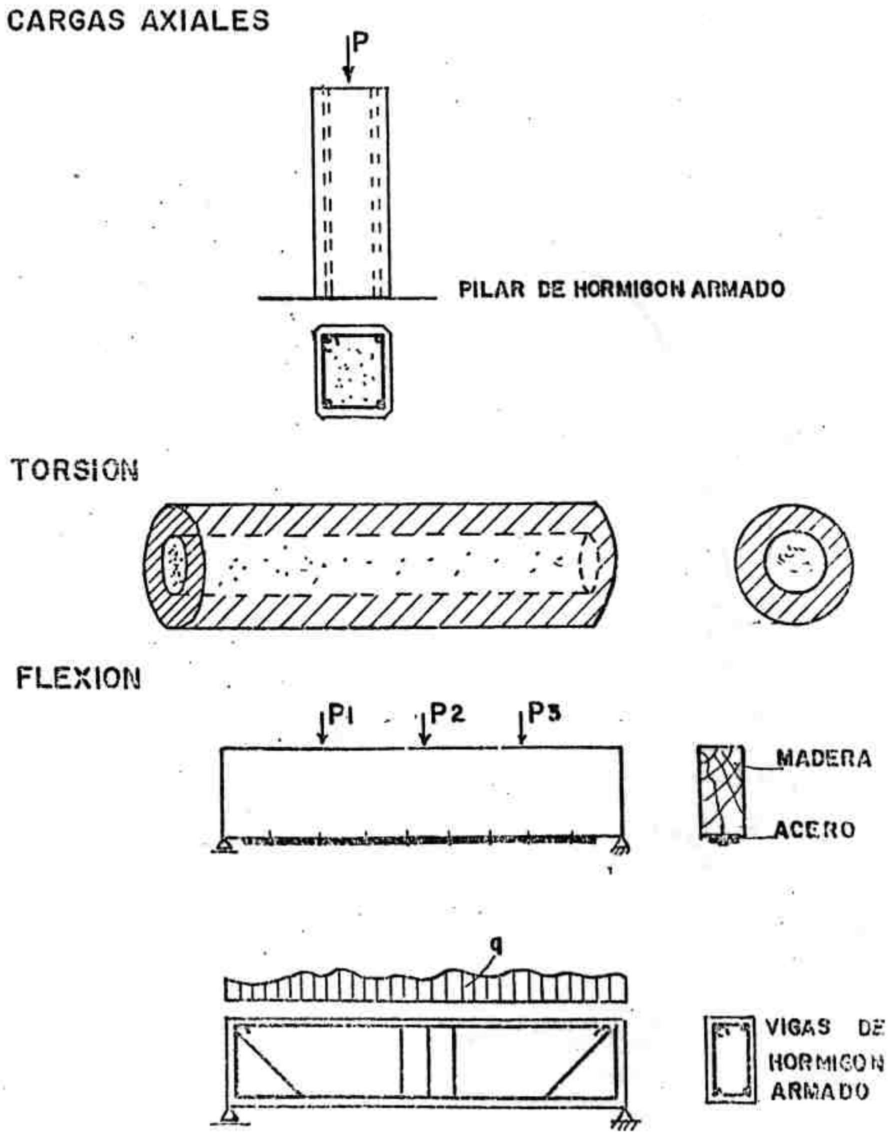
Vigas de 2 materiales

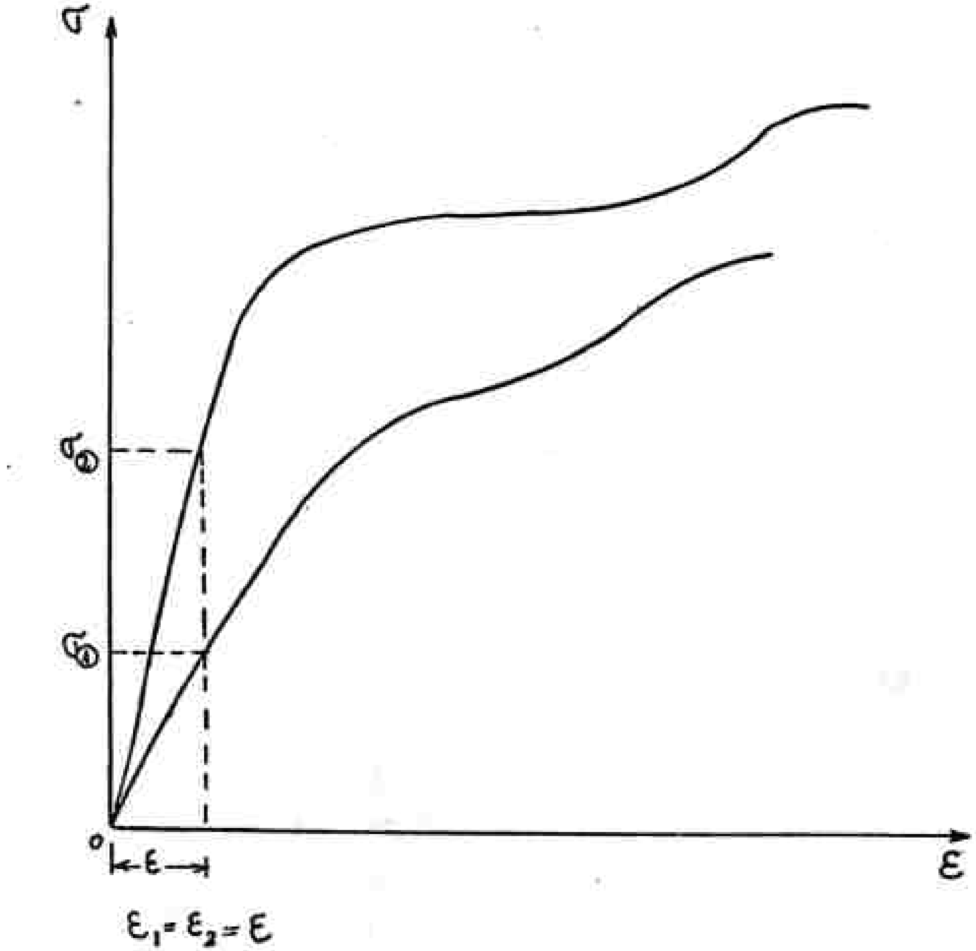
Clase 17

Método general, Método de la Sección Transformada, Tensiones de Corte, Flexión Compuesta, Desplazamientos.

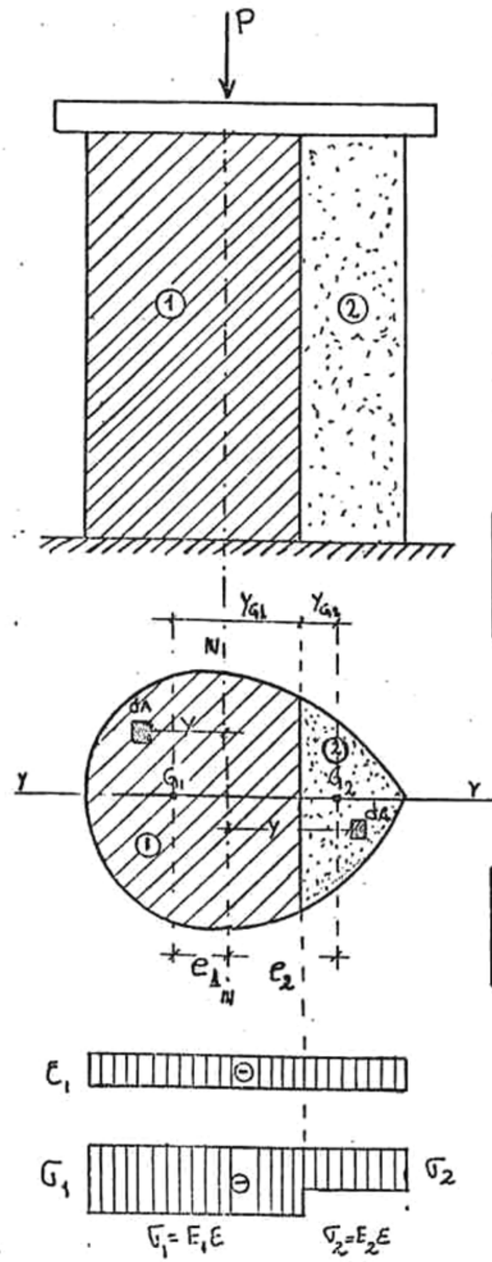
BARRAS DE DOS MATERIALES



En vigas de dos materiales, en aquellos puntos en donde se le obligan a ambos materiales a trabajar juntos, a igual deformación corresponde tensiones diferentes



CASO1: CARGAS PARALELAS AL EJE



$$\int_{(1)} \sigma_1 \, dA + \int_{(2)} \sigma_2 \, dA = P \qquad (1)$$

$$\int_{(1)} \sigma_1 y \, dA + \int_{(2)} \sigma_2 y \, dA = 0 \qquad (2)$$

$$\varepsilon_1 = \varepsilon_2 = \varepsilon = \frac{\sigma_1}{E} = \frac{\sigma_2}{E} \qquad (3)$$

$$e_1 + e_2 = Y_{G1} + Y_{G2} \qquad (4)$$

DE (1) $\sigma_1 A_1 + \sigma_2 A_2 = P$

DE (3) $\sigma_2 = \frac{E_2}{E_1}\sigma_1$

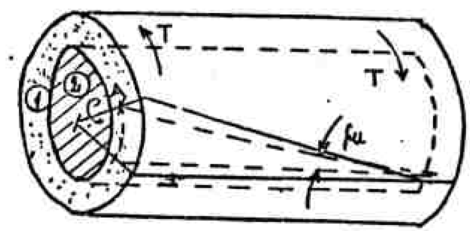
$$\sigma_1 = \frac{P E_1}{A_1 E_1 + A_2 E_2} \qquad \sigma_2 = \frac{P E_2}{A_1 E_1 + A_2 E_2}$$

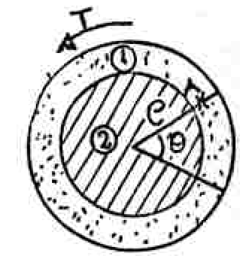
DE (2) $\sigma_1 M_{EST1} = \sigma_2 M_{EST2}$

DE (4) $e_1 + e_2 = Y_{G1} + Y_{G2}$

$$e_1 = \frac{(Y_{G1} + Y_{G2}) E_2 A_2}{A_1 E_1 + A_2 E_2} \qquad e_2 = \frac{(Y_{G1} + Y_{G2}) A_1 E_1}{A_1 E_1 + A_2 E_2}$$

CASO 2:TORSION





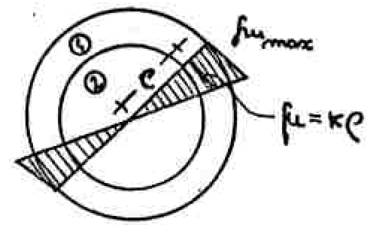
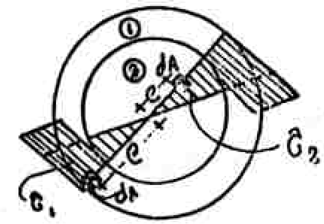


DIAGRAMA DE γ



DIAAGRAMA DE τ

$$\int_{1} \rho\, \tau_1\, dA + \int_{2} \rho\, \tau_2\, dA = T \qquad (1)$$

$$\gamma = K\rho$$

$$\tau_1 = G_1 \gamma = G_1 K \rho$$

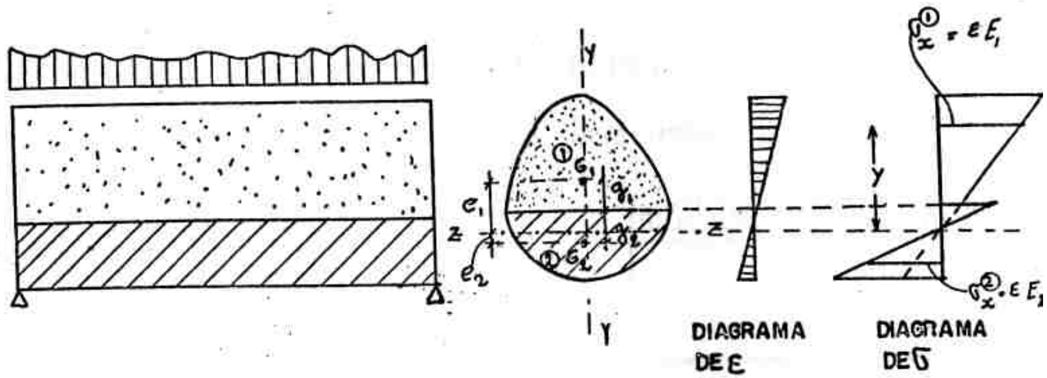
$$\tau_2 = G_2 \gamma = G_2 K \rho \qquad (2)$$

$$K = \frac{T}{G_1 I_{P_1} + G_2 I_{P_2}}$$

$$\tau_1 = \frac{T G_1 \rho}{G_1 I_{P_1} + G_2 I_{P_2}}$$

$$\tau_2 = \frac{T G_2 \rho}{G_1 I_{P_1} + G_2 I_{P_2}}$$

CASO3: FLEXIÓN



$$\int_{①} \sigma_x^{(1)}\, dA + \int_{②} \sigma_x^{(2)}\, dA = 0 \qquad ①$$

$$\int_{①} \sigma_x^{(1)}\, y\, dA + \int_{②} \sigma_x^{(2)}\, y\, dA = M \qquad ②$$

$$\varepsilon = \frac{y}{\rho}$$

$$\sigma_x^{(1)} = \varepsilon E_1 = \frac{y}{\rho} E_1 \qquad \sigma_x^{(2)} = \varepsilon E = \frac{y}{\rho} E_2 \qquad ③$$

$$e_1 + e_2 = g_1 + g_2 \qquad ④$$

DE ② y ③

$$\frac{E_1}{\rho}\int_{①} y^2\, dA + \frac{E_2}{\rho}\int_{②} y^2\, dA = M$$

$$\frac{1}{\rho} = \frac{M}{E_1 I_1 + E_2 I_2}$$

$$\sigma_x^{(1)} = \frac{M E_1 y}{E_1 I_1 + E_2 I_2} \qquad \sigma_x^{(2)} = \frac{M E_2 y}{E_1 I_1 + E_2 I_2}$$

DE ① y ③

$$\frac{E_1}{\rho}\int_{①} y\, dA = \frac{E_2}{\rho}\int_{②} y\, dA$$

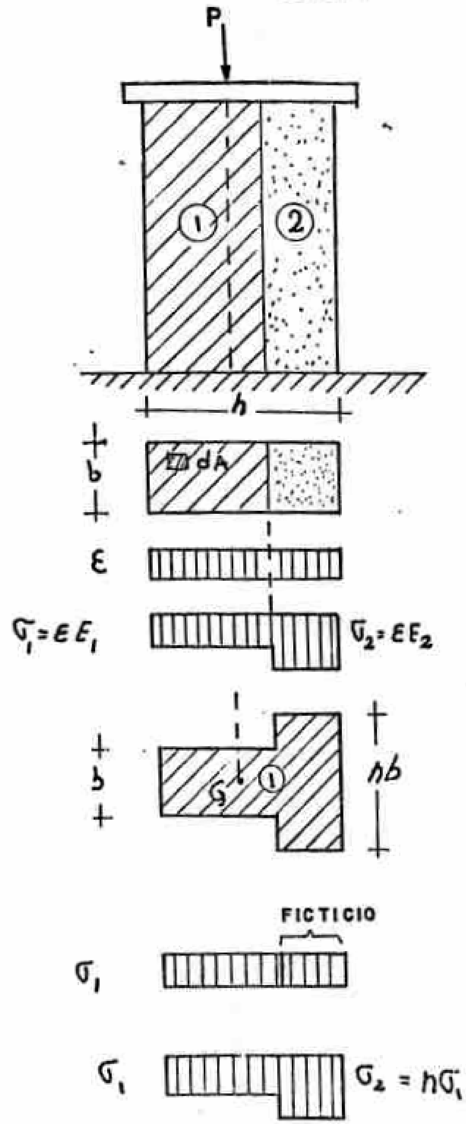
$$E_1 A_1 e_1 = E_2 A_2 e_2$$

y con ④

$$e_1 = \frac{(g_1+g_2) E_2 A_2}{E_1 A_1 + E_2 A_2} \qquad e_2 = \frac{(g_1+g_2) A_1 E_1}{E_1 A_1 + E_2 A_2}$$

METODO DE LA SECCIÓN TRANSFORMADA

CARGAS PARALELAS AL EJE



$$\sigma_1 = \frac{P E_1}{A_1 E_1 + A_2 E_2} = \frac{P}{A_1 + n A_2} = \frac{P}{A_{TRANS.}}$$

$$\frac{E_2}{E_1} = n$$

$$A_2 = b\,h_2$$

$$n A_2 = (n b)\,h_2$$

$$dF_1 = \sigma_1 dA \qquad dF_2 = \sigma_2 dA$$

$$dF_1 = \varepsilon E_1 dA \qquad dF_2 = \varepsilon E_2 dA$$

$$\frac{E_2}{E_1} = n = \frac{\sigma_2}{\sigma_1}$$

$$dF_2 = \varepsilon n E_1 dA$$

$$dF_2 = \varepsilon E_1 (n\,dA)$$

AREA EQUIVALENTE A (1)

$$\int_{(1)} \sigma_1 y\,dA = \int_{(2)} \sigma_2 y\,dA$$

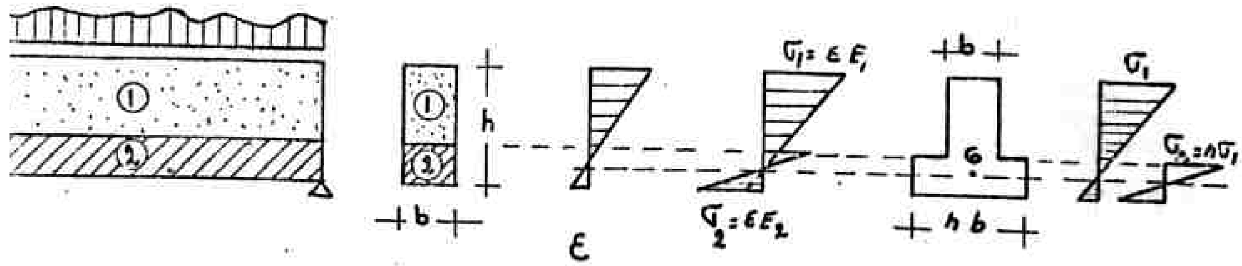
$$\frac{\sigma_2}{\sigma_1} = \frac{E_2}{E_1} = n$$

$$\int_{(1)} y\,dA = \int_{(2)} y\,(n\,dA)$$

SE DEBE ALTERAR EL ANCHO PORQUE LA DISTANCIA "Y" NO DEBE ALTERARSE

FLEXIÓN:

METODO DE LA SECCIÓN TRANSFORMADA



$$\sigma_1 = \frac{M E_1 y}{I_{z_1} E_1 + I_{z_2} E_2} = \frac{M y}{I_{z_1} + n I_{z_2}} = \frac{M y}{I_{TRANS}}$$

$$\frac{E_2}{E_1} = n = \frac{\sigma_2}{\sigma_1} \qquad I_{z_2} = \frac{b h^3}{\alpha} \qquad n I_{z_2} = (nb)\frac{h^3}{\alpha}$$

$$dF_1 = \sigma_1 dA = \frac{E_1 y dA}{\rho}$$

$$dF_2 = \sigma_2 dA = \frac{E_2 y dA}{\rho} = \frac{E_1 y}{\rho}(n dA)$$

AREA EQUIVALENTE AL MAT. (1)

$$\frac{E_1}{\rho}\int_{1} y\, dA = \frac{E_2}{\rho}\int_{2} y\, dA$$

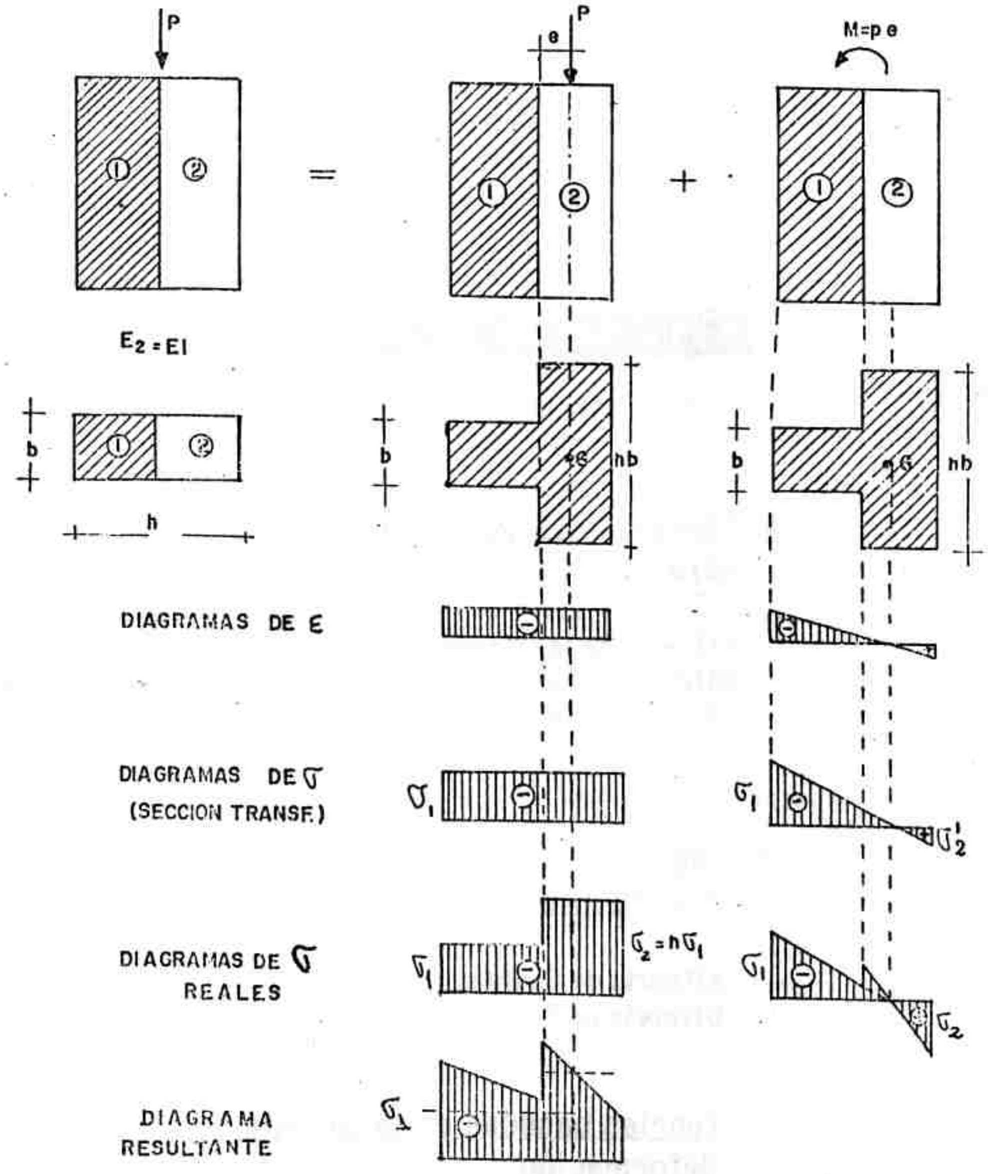
$$\int_{1} y\, dA = \int_{2} y\,(n dA) \rightarrow$$

El Eje Neutro no cambiará si cada elemento del cuerpo 2 es multiplicado por un factor "n", siempre que no se altere la distancia "y" de cada uno de ellos al mismo eje. O sea: SOLO SE PUEDE VARIAR EL ANCHO

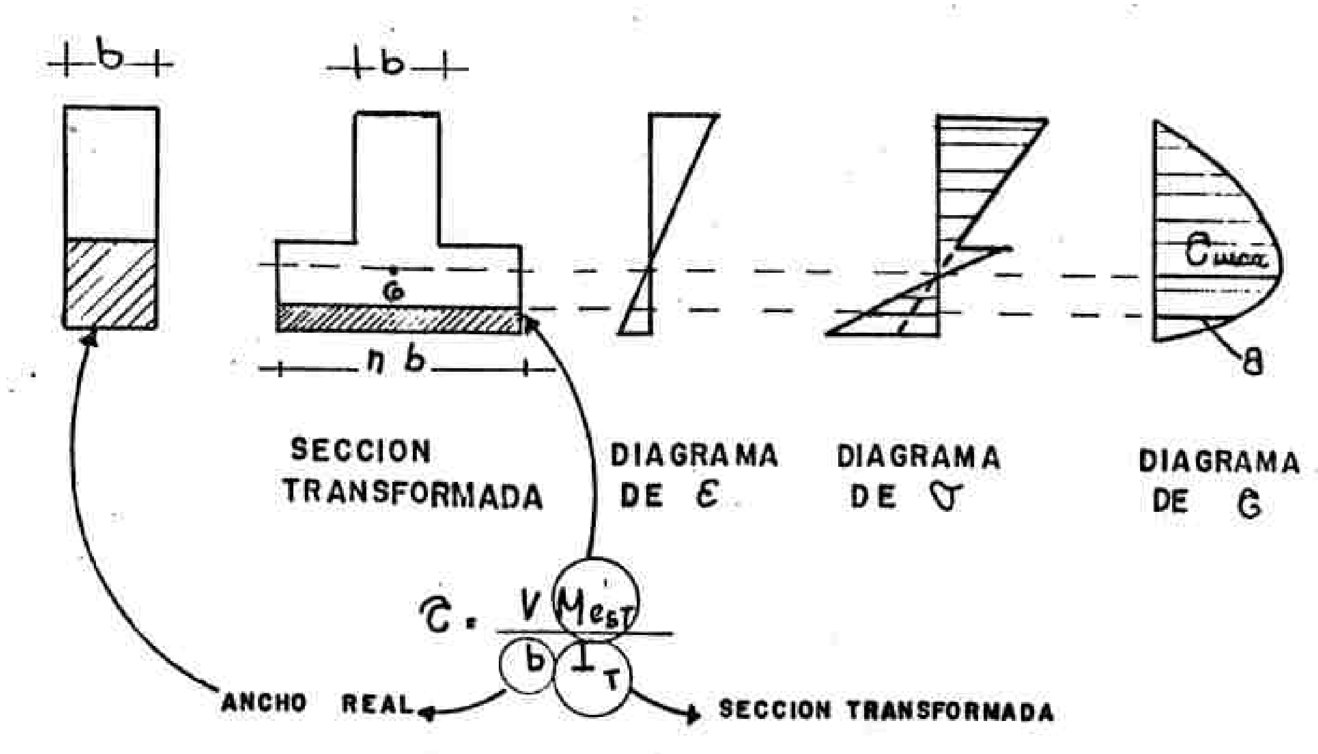
La resistencia de la Sección Transformada es la misma que la original

$$M = \int_A \sigma . y . dA = \int_1 \sigma_1 . y . dA + \int_2 \eta . \sigma_1 . y . dA = \frac{1}{\rho} . (E_1 . I_1 + E_2 . I_2)$$

FLEXION COMPUESTA



TENSION DE CORTE EN LA FLEXIÓN



DEFORMACIONES

Las deformaciones y los desplazamientos se pueden determinar usando la Sección Transformada. Esta sección representa la sección recta de un elemento hecho de un material homogéneo que se deforma de la misma manera que el elemento compuesto

Próxima Clase: Estado plano de Tensiones

Fin