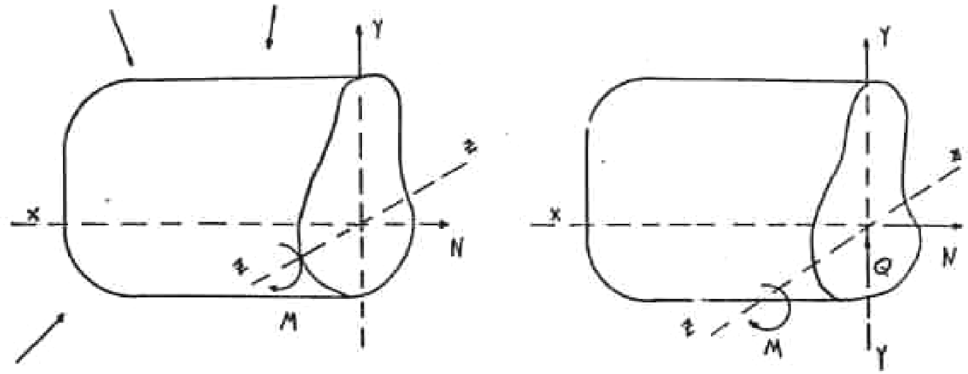
Flexión Compuesta

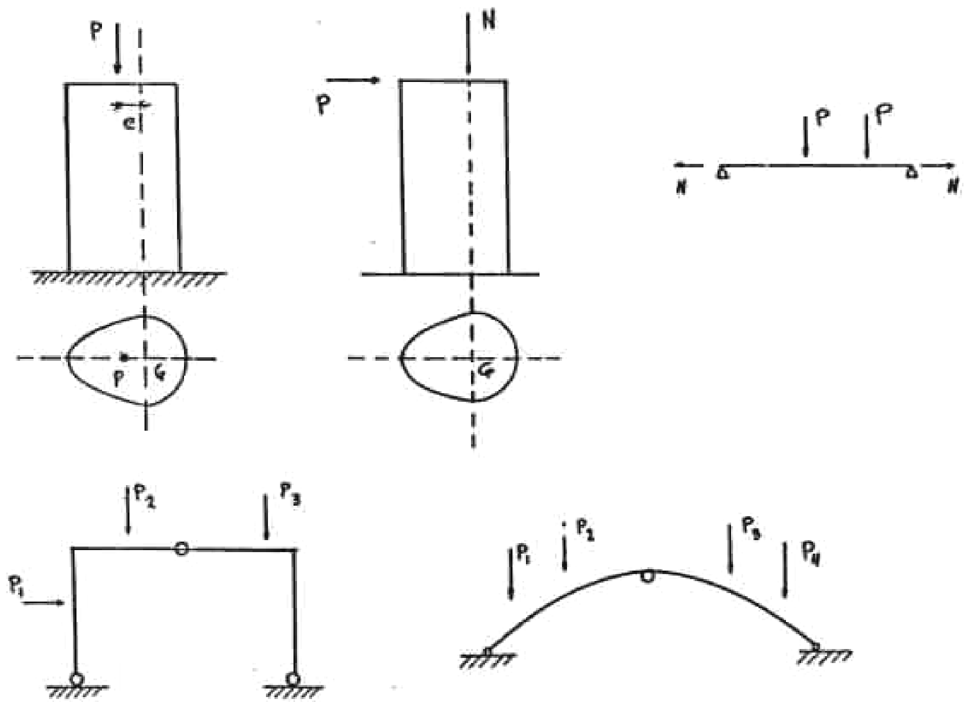
Clase 16

Piezas Cortas

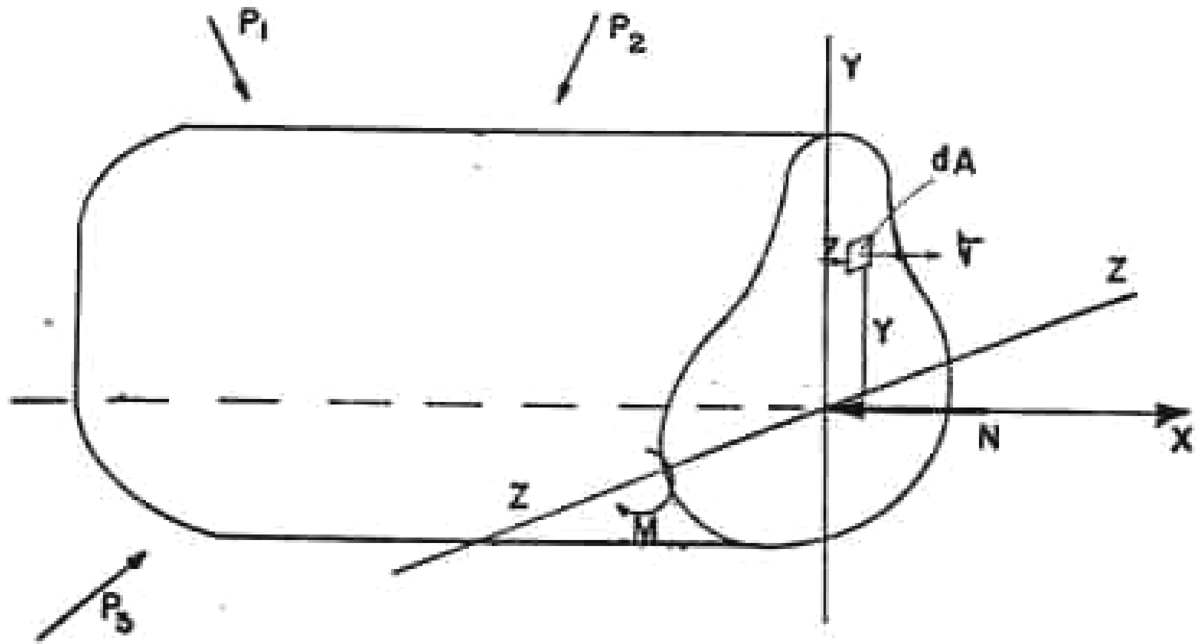
FLEXION COMPUESTA



CASOS TIPICOS



DEDUCCION DE LA FÓRMULA



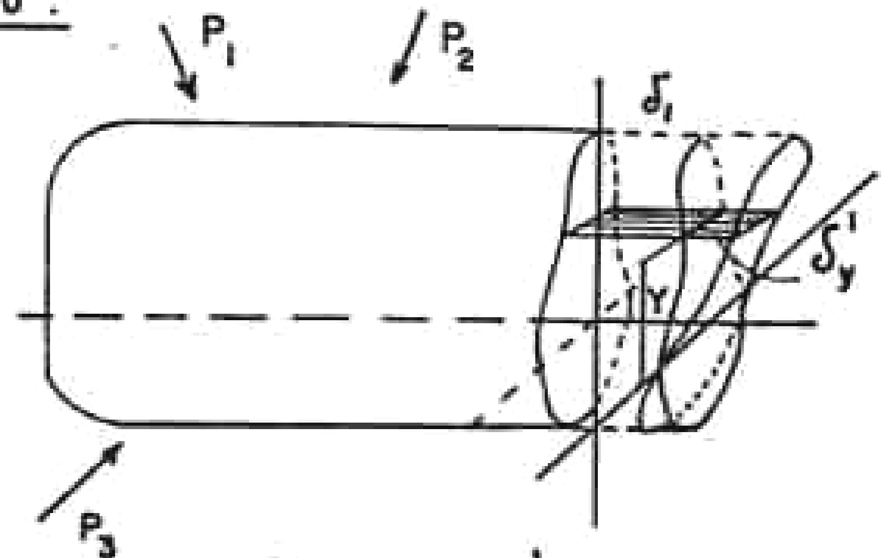
1er Paso : a) $dF = \sigma \, dA$

b) $\int_A \sigma \, dA = N$

$\int \sigma Y \, dA = M$ $\int \sigma Z \, dA = 0$

DEDUCCION DE LA FÓRMULA

2º Paso :



$$\delta_y = \delta_1 + \delta_y'$$

$$\varepsilon_y = \varepsilon_1 + \varepsilon_y \qquad \varepsilon_1 = A \qquad \varepsilon_y = BY$$

$$\sigma = \varepsilon_y E = E.A + E\,BY = a + bY$$

3º Paso

$$\int_A (a+by)\,dA = N \qquad a\int_A dA + b\int_A y\,dA = N \qquad (1)$$

$$\int_A (a+by)\,y\,dA = M \qquad a\int_A y\,dA + b\int_A y^2\,dA = M \qquad (2)$$

$$\int_A (a+by)\,z\,dA = 0 \qquad a\int z\,dA + b\int yz\,dA = 0 \qquad (3)$$

DE (1) $\quad a\,A + b\,M_{S_{LN}} = N \qquad M_{S_{LN}} = 0 \qquad a = \dfrac{N}{A}$

DE (2) $\quad a\,M_{S_{LN}} + b\,I_z = M \qquad b = \dfrac{M}{I_z}$

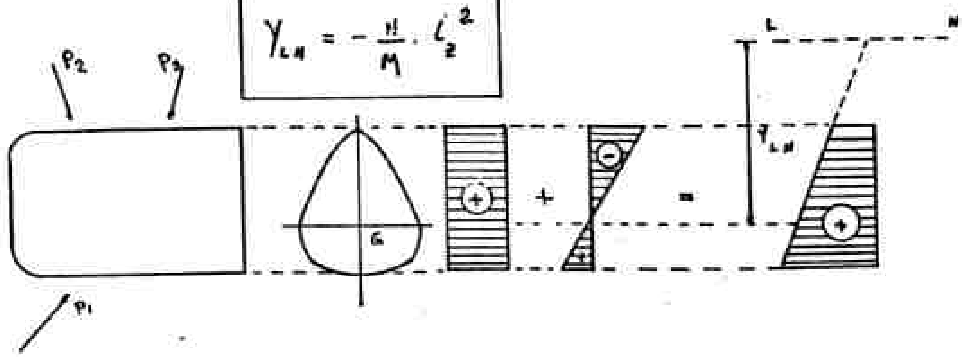
$$\boxed{\sigma = \frac{N}{A} + \frac{M}{I_z}\,y}$$

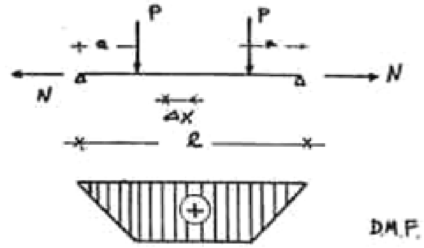
Posición de la LN

$$\frac{N}{A} + \frac{M}{I_z}\cdot Y_{LN} = 0$$

$$Y_{LN} = -\frac{N}{M}\cdot\frac{I_z}{A}$$

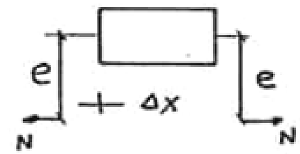
$$\boxed{Y_{LN} = -\frac{N}{M}\cdot i_z^{2}}$$





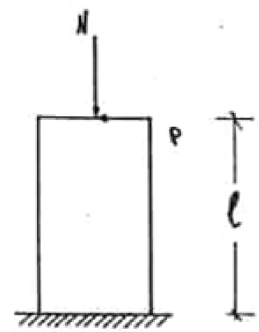
$$M = N \cdot e$$

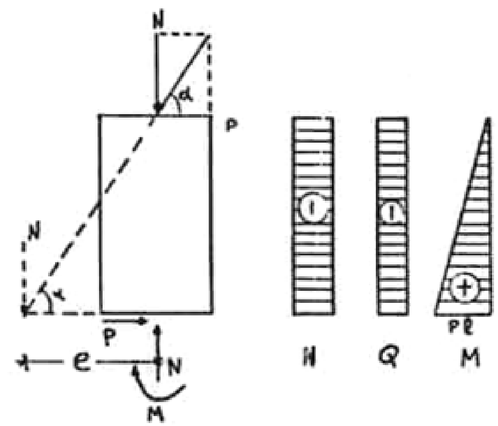
$$e = \frac{M}{N}$$



e = EXCENTRICIDAD - POSICION DEL CENTRO DE PRESION

CENTRO DE PRESIÓN





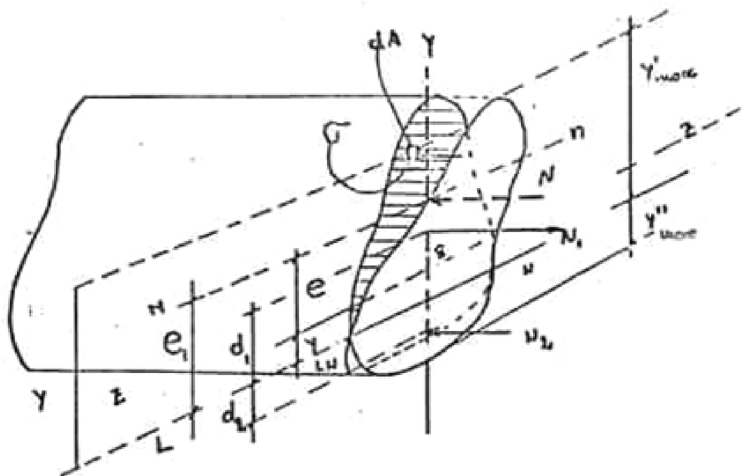
$$e = \frac{M}{N} = \frac{P\ell}{N}$$

DE LA FIG.

$$e = \frac{\ell}{tg\alpha} = \frac{\ell}{\frac{N}{P}} = \frac{P\ell}{N} = \frac{M}{N}$$

RELACION ENTRE LA POSICION DE LA L.N. Y LA EXENTRICIDAD

$$Y_{LN} = -\frac{N}{M}\, i_z^2 \qquad \boxed{Y_{LN} = -\frac{1}{e}\, i_z^2} \qquad \boxed{Y_{LN} \cdot e = -i_z^2}$$



$$\int_A \sigma \, dA = N_1 - N_2 = N$$

$$N_1 = \int_0^{Y'_{max}} \sigma \, dA \qquad N_2 = \int_0^{-Y''_{max}} \sigma \, dA$$

$$M_1 = \int_0^{Y'_{max}} \sigma \, y \, dA \qquad M_2 = \int_0^{-Y''} \sigma \, y \, dA$$

$$\boxed{\sigma = \frac{M y}{I'_{LN}} = \frac{N e_1 Y}{I_{LN}}}$$

$$\frac{\sigma}{y} \int_0^{Y'_{max}} y \, dA - \frac{\sigma}{y} \int_0^{-Y''_{max}} y \, dA = N$$

$$\boxed{\sigma = \frac{N y}{Ms_{LN}}}$$

$$e_1 = \frac{M_1 + M_2}{N} = \frac{\int_0^{Y'_{max}} \sigma y \, dA + \int_0^{-Y''_{max}} \sigma y \, dA}{\int_0^{Y'_{max}} \sigma \, dA - \int_0^{-Y''_{max}} \sigma \, dA} = \frac{\frac{\sigma}{y}\left[I'_{LN} + I''_{LN} \right]}{\frac{\sigma}{y}\left[M'_{sLN} - M''_{sLN} \right]} = \frac{I_{LN}}{M_{sLN}}$$

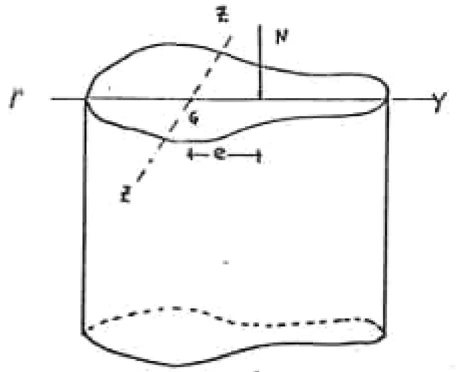
POR OTRO LADO

$$e_1 = e + Y_{LN} = e + \frac{i_z^2}{e} = e + \frac{I_z}{Ae} = \frac{Ae^2 + I_z}{Ae} = \frac{I_{n-n}}{M_{s\,n-n}} = \frac{I_{LN}}{M_{s\,LN}}$$

Si $y''_{max} = 0 \qquad N_2 = 0 \qquad N_1 = N$

$$\int_A \sigma \, dA = N \qquad e = \frac{M}{N}$$

$$e_1 = \frac{I_{zN}}{M_{s_{zN}}}$$

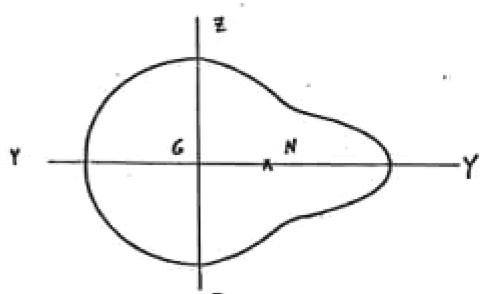


$$\int_A \sigma \, dA = N$$

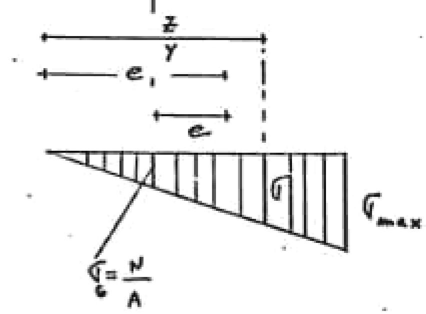
$$\frac{\sigma}{y} \int_A y \, dA = N$$

$$\frac{\sigma}{y} M_{s_{zN}} = N$$

$$\boxed{\sigma = \frac{N y}{M_{s_{zN}}}} \qquad \boxed{\sigma = \frac{N e_1 y}{I_{zN}}}$$



$$\sigma_G = \frac{N(e_1 - e)}{A(e_1 - e)} = \frac{N}{A}$$



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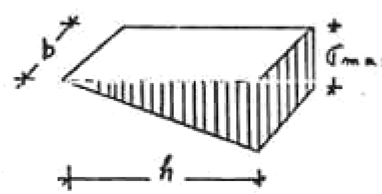
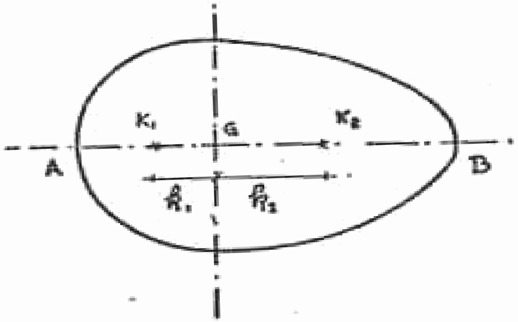
SECCION RECTANGULAR

$$\sigma_{max} = \frac{N \frac{h}{2}}{b h \cdot \frac{h}{2}} = \frac{2N}{bh}$$

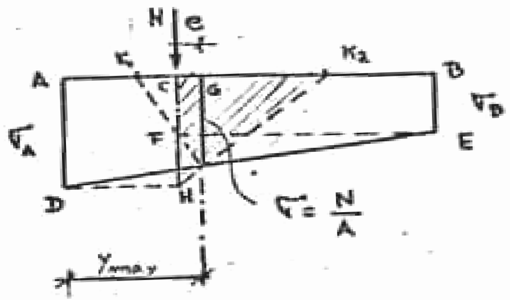


PROCESO GRAFICO PARA HALLAR LAS TENSIONES

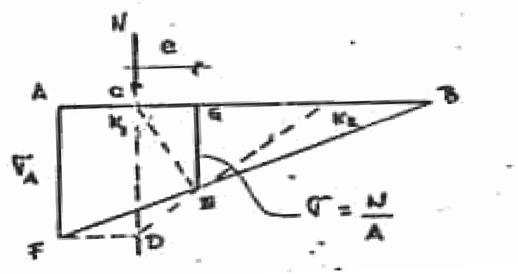


$$\sigma_A = \frac{\sigma.(e+k_2)}{k_2} = \frac{N}{A}.\frac{(e+k_2)}{\frac{i_z{}^2}{y_{\max}}} = \frac{N}{A}.\frac{(e+k_2)}{\frac{I_z}{A.y_{\max}}} = N.\frac{(e+k_2)}{W}$$

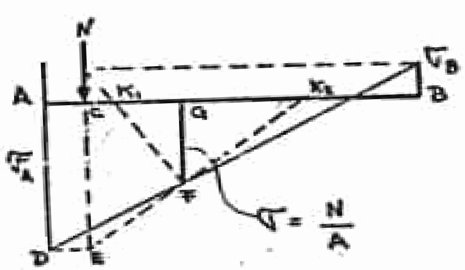
(Fórmula del momento nuclear)



1º Caso: e < K_1

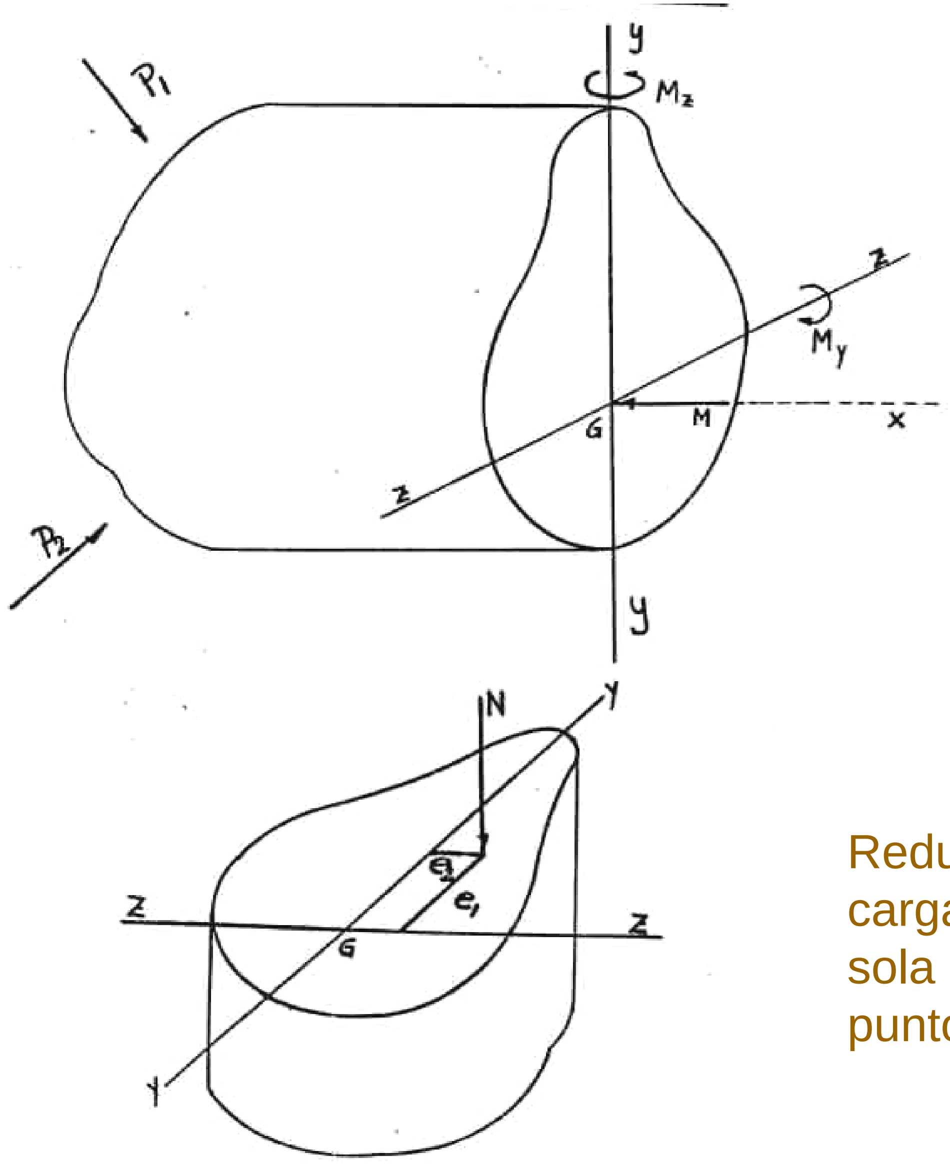


2º Caso: e = K_1



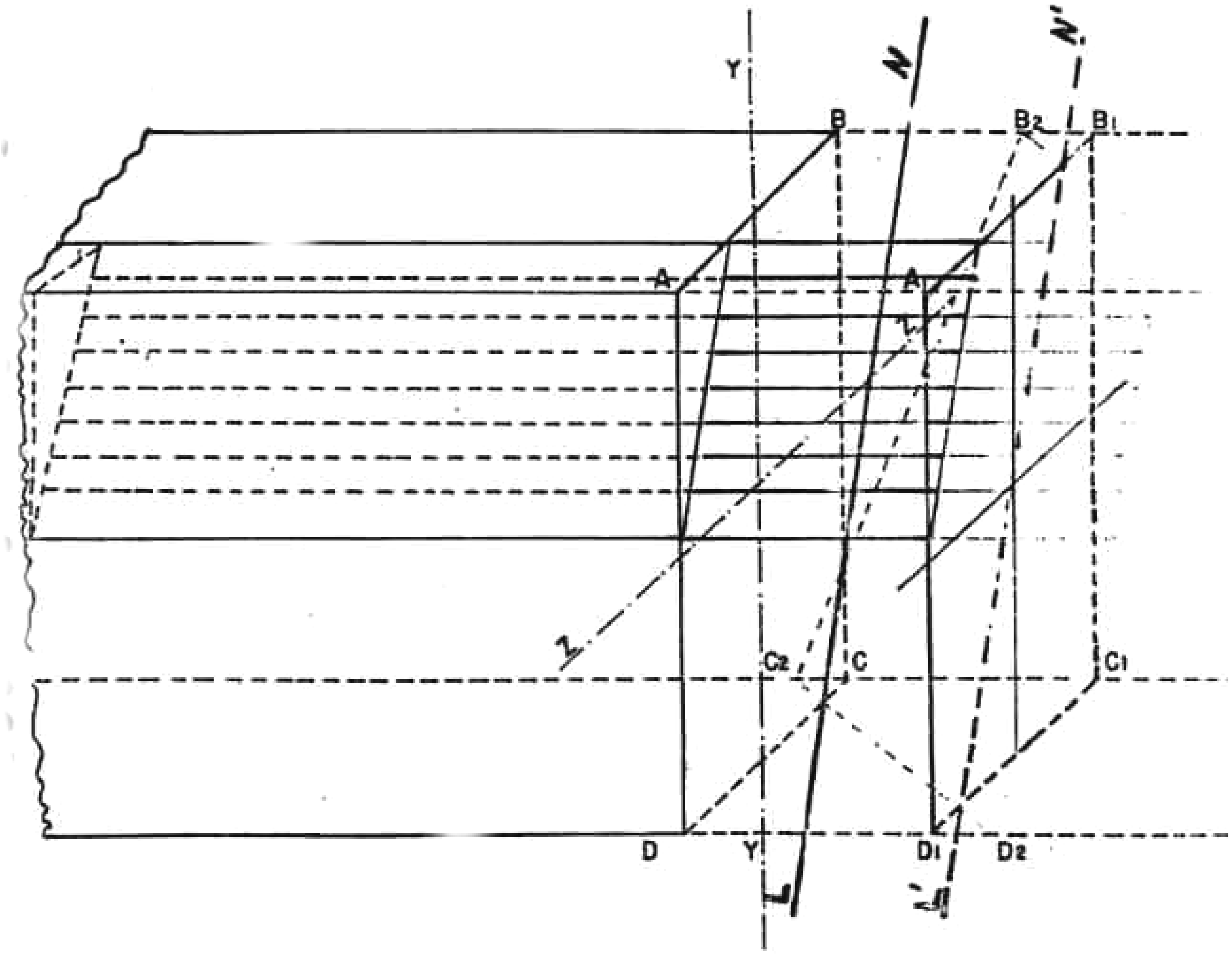
3º Caso: e > K_1

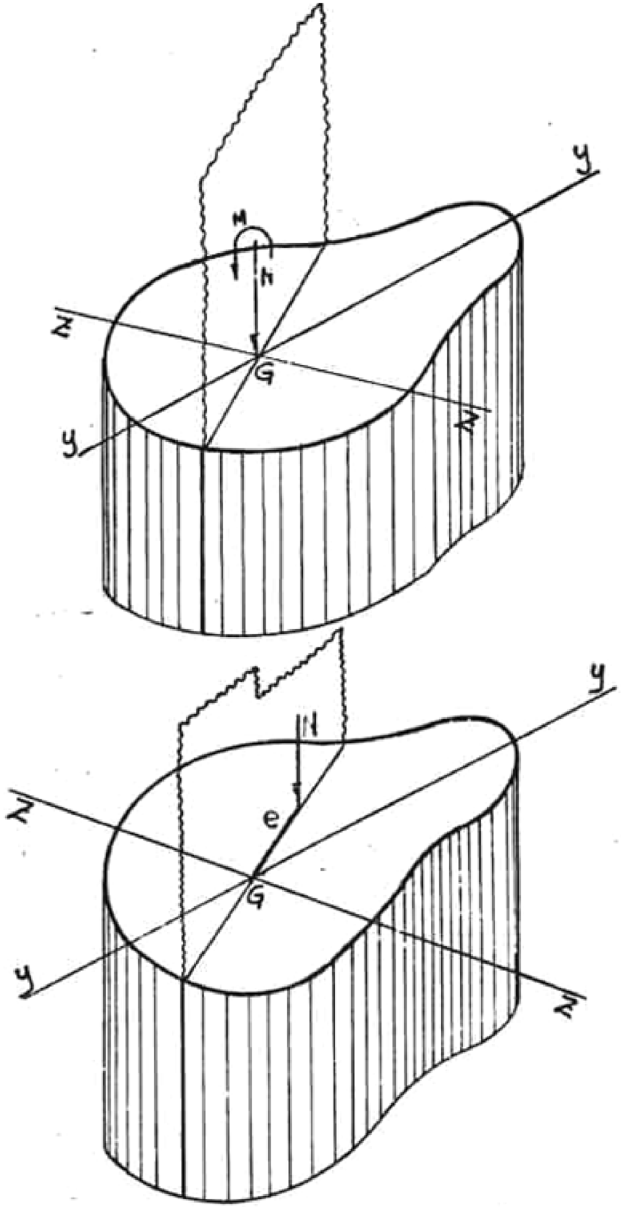
FLEXIÓN OBLICUA COMPUESTA

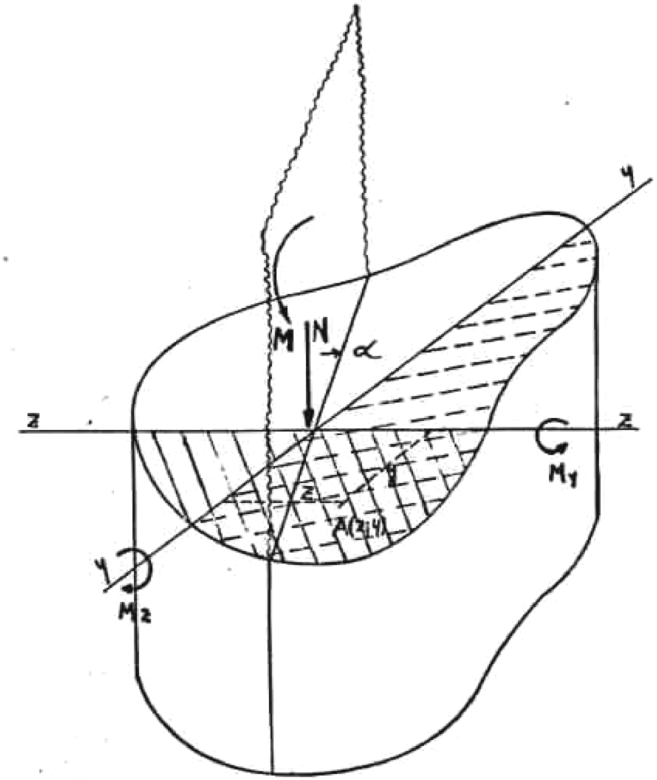


Reducción de todas las cargas externas a una sola **N** aplicada en un punto (e1;e2)

FLEXION OBLICUA COMPUESTA



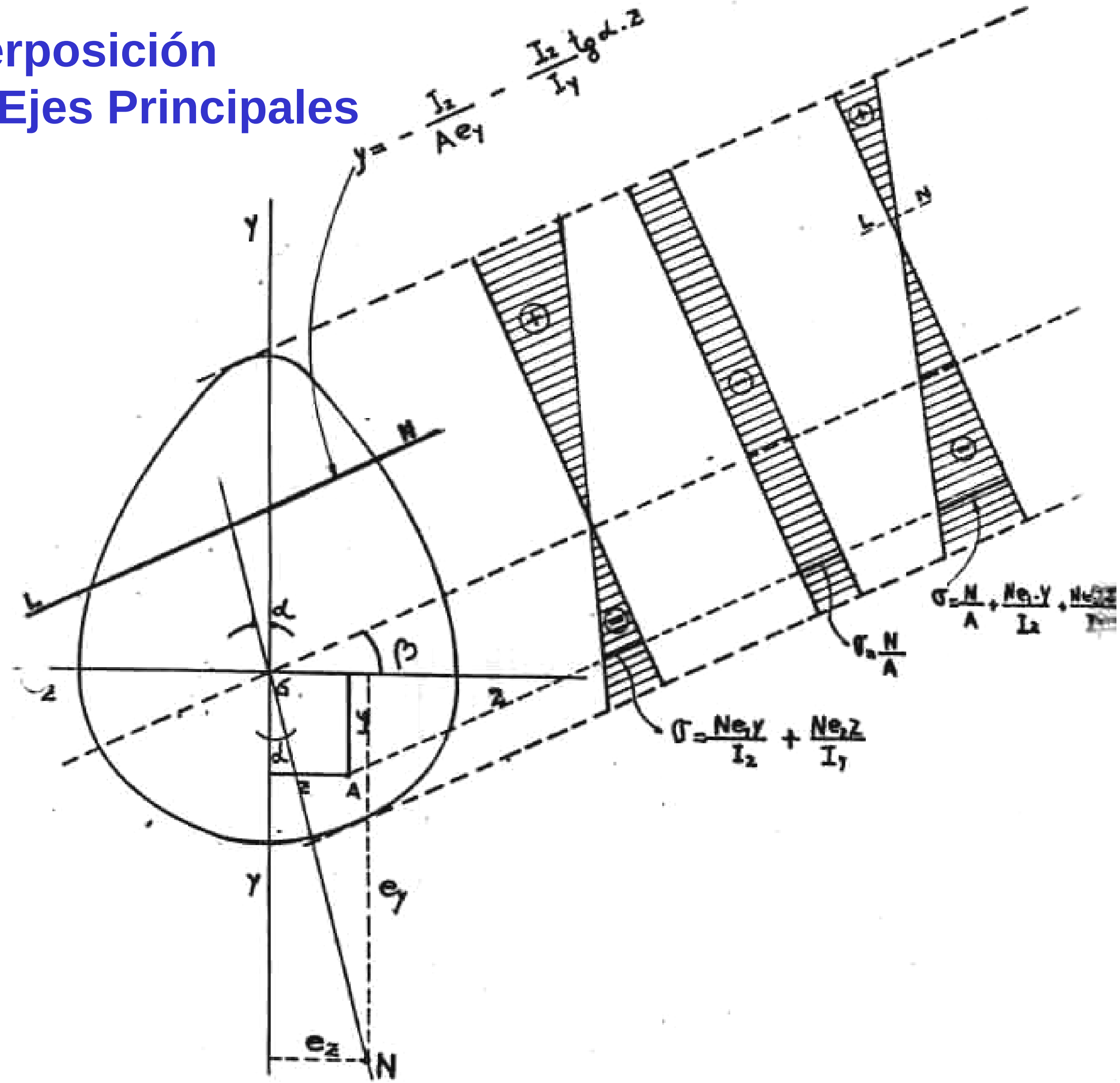




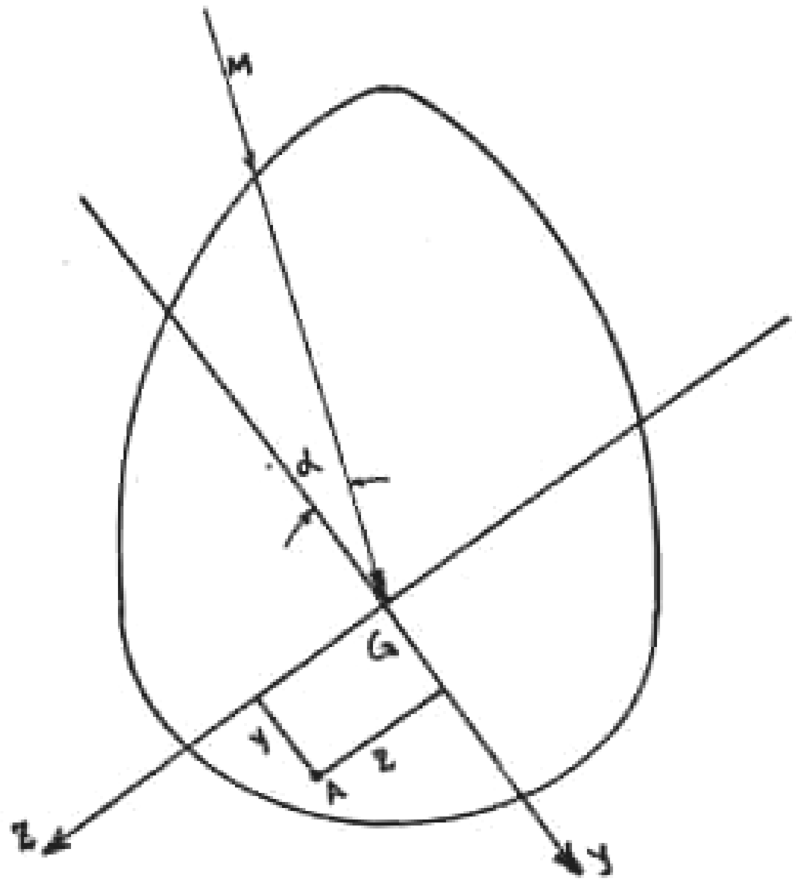
$$\boxed{\sigma = \frac{N}{A} \pm \frac{M_y\, y}{I_z} \pm \frac{M_z\, z}{I_y}}$$

$$y = -\frac{N}{A} \cdot \frac{I_z}{M_y} - \frac{I_z}{I_y}\, tg\alpha \cdot z$$

Método de Superposición
Para el caso de Ejes Principales

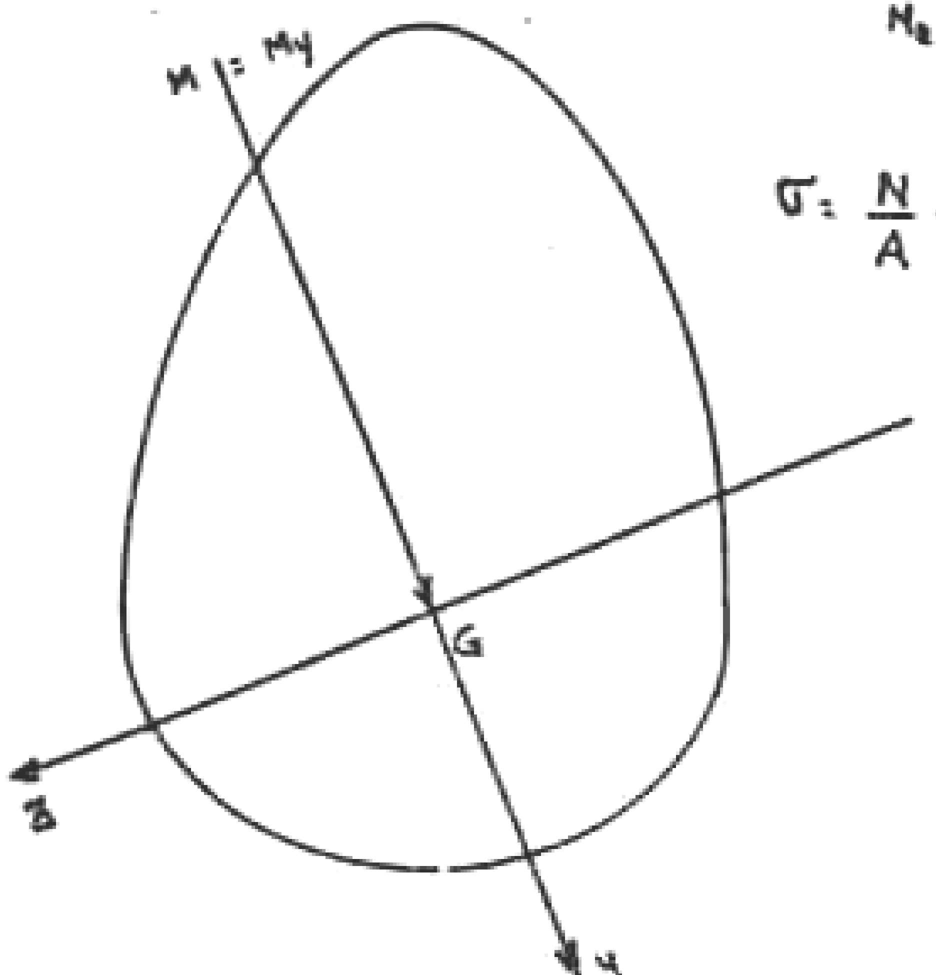


FORMULA REFERIDA A EJES CENTROIDALES CUALESQUIERA



$$\sigma = \frac{N}{A} + \frac{M_z I_z - M_y I_{zy}}{I_z I_y - I_{zy}^2} z + \frac{M_y I_y - M_z I_{zy}}{I_z I_y - I_{zy}^2} y$$

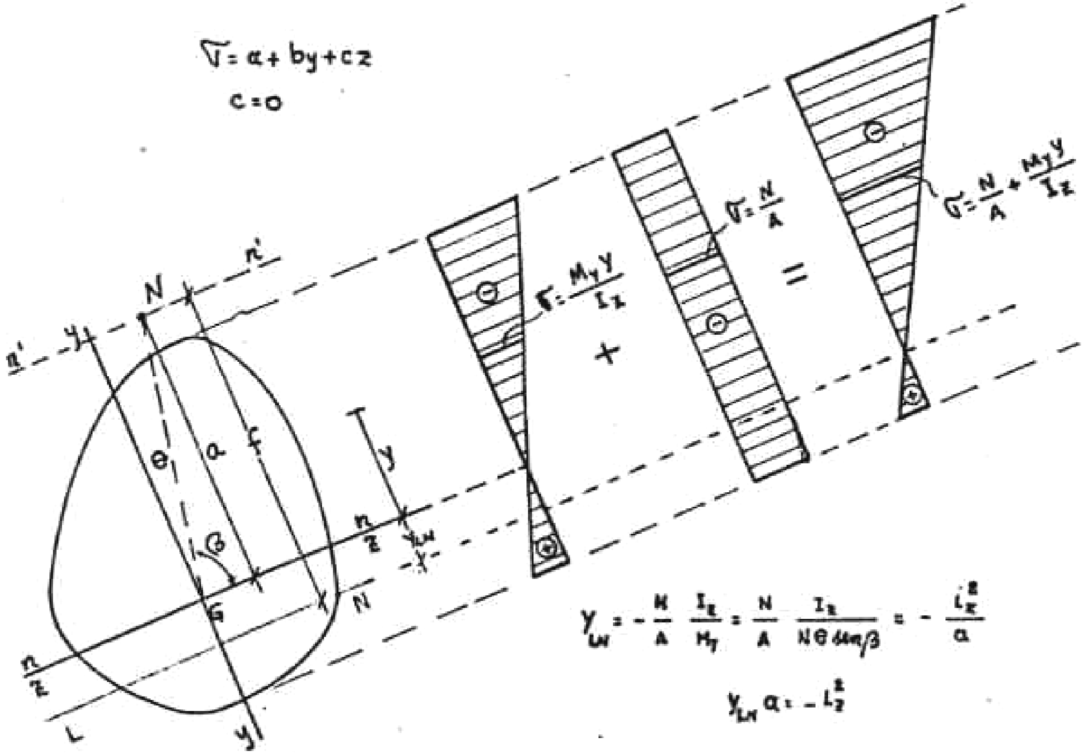
FORMULA REFERIDA A EJES CENTROIDALES, UNO DE ELLOS COINCIDENTE CON EL PLANO DE CARGA



$$M_z = 0$$

$$\sigma = \frac{N}{A} + \frac{M\left(I_{zy}\, z + I_y\, y\right)}{I_z I_y - I_{zy}^2}$$

SISTEMA DE EJES CENTROIDALES, UNO DE ELLOS PARALELO A LA LINEA NEUTRA



$$y_{LN} = -\frac{N}{A}\,\frac{I_z}{M_y} = \frac{N}{A}\,\frac{I_z}{N\,e\,\operatorname{sen}\beta} = -\frac{i_z^2}{a}$$

$$y_{LN}\,a = -i_z^2$$

$$f = a + y_{LN}$$

$$f = a + \frac{i_z^2}{a} = a + \frac{I_z}{Aa}$$

$$f = \frac{Aa^2 + I_z}{Aa}$$

$$f = \frac{I_{n'n'}}{S_{n'n'}}$$

$$f = a + y_{LN}$$

$$f = \frac{i_z^2}{y_{LN}} + y_{LN} = \frac{I_z}{A\,y_{LN}} + y_{LN}$$

$$f = \frac{I_z + A\,y_{LN}^2}{A\,y_{LN}}$$

$$f = \frac{I_{LN}}{S_{LN}}$$

Próxima Clase: Vigas de 2 materiales

Fin