# Flexión Oblicua

Clase 14

Tensiones

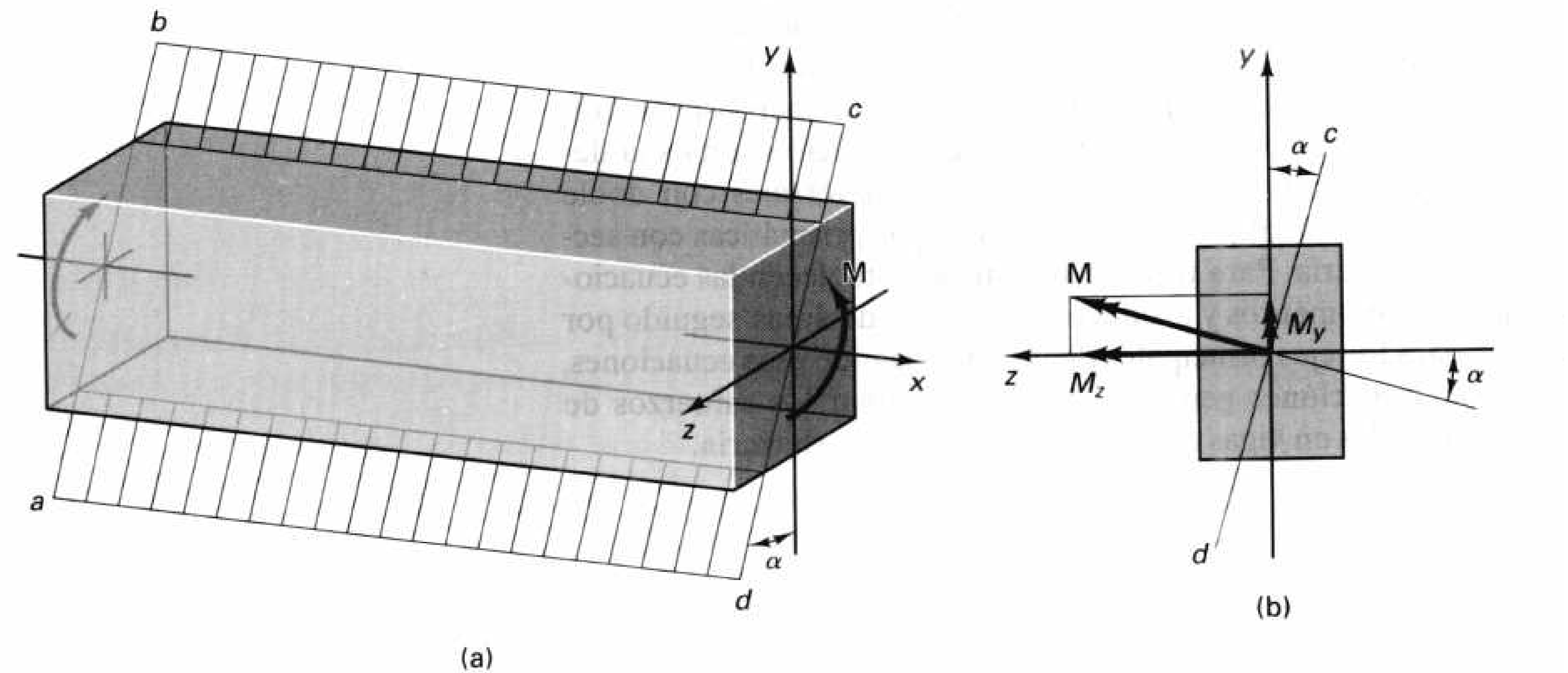



**Fig. 9-1** Flexión asimétrica de una viga con sección transversal doblemente simétrica.

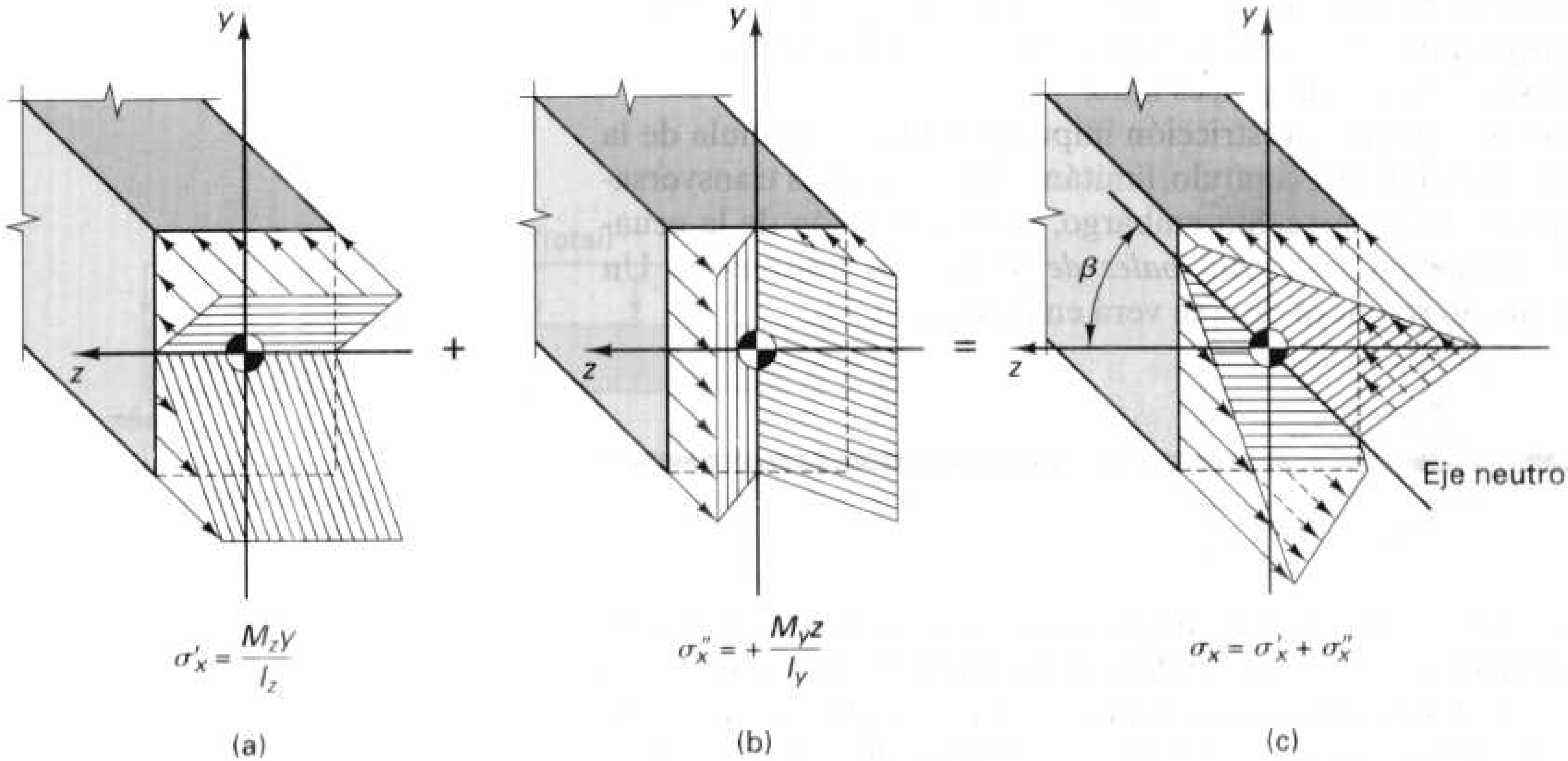
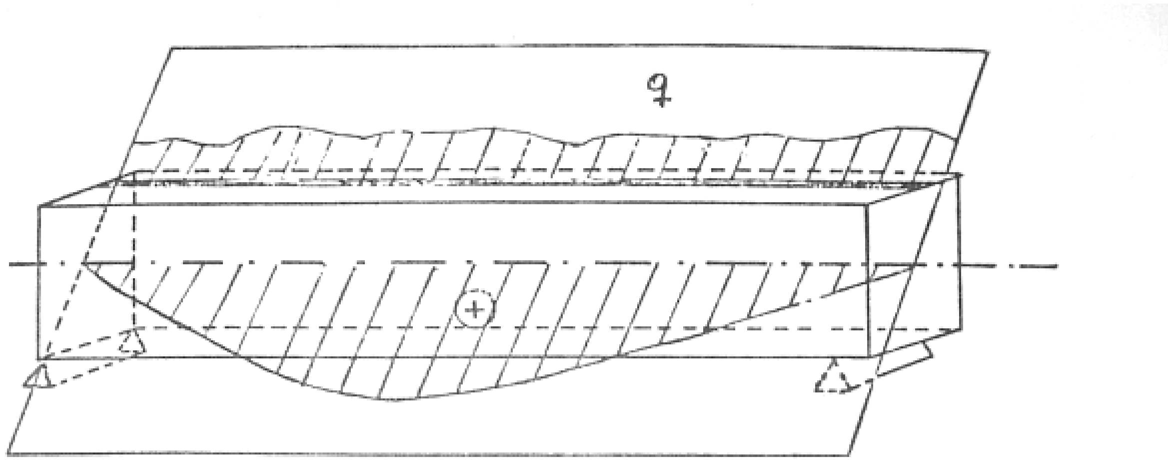
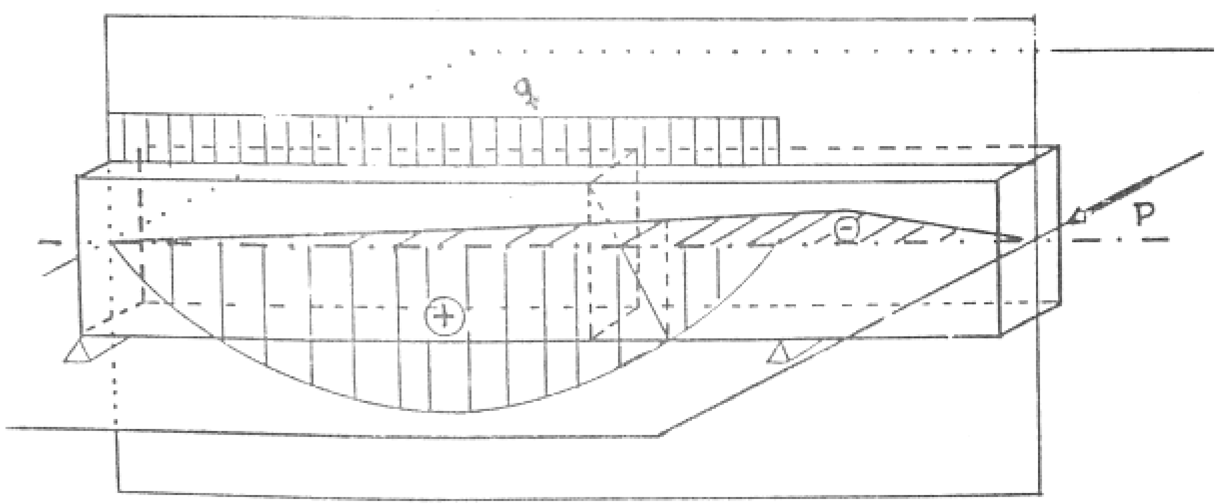



**Fig. 9-2** Superposición de esfuerzos elásticos de flexión.

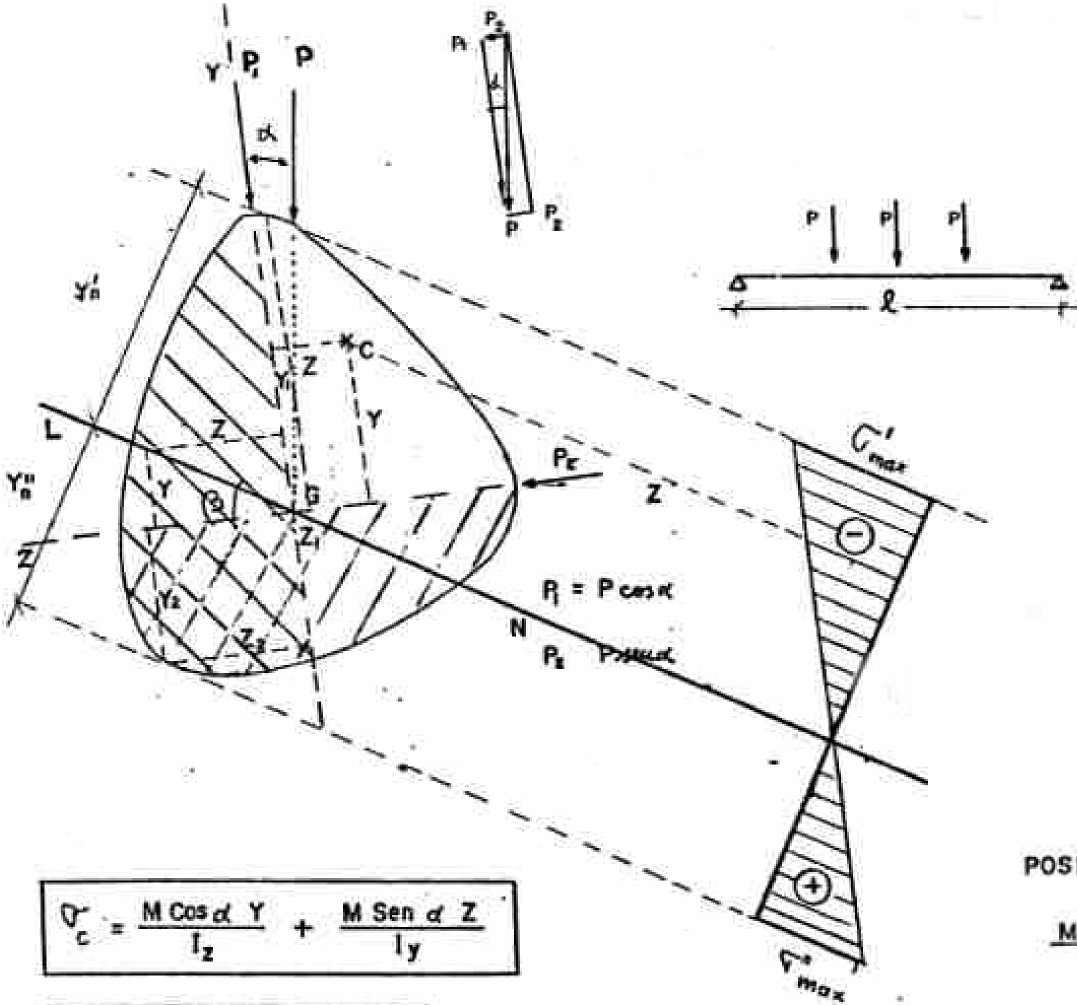
# FLEXION DESVIADA




FLEXION DOBLE



# FLEXIÓN OBLICUA




$$\sigma_c = \frac{M \cos\alpha \ Y}{I_z} + \frac{M \operatorname{Sen}\alpha \ Z}{I_y}$$

$$\sigma_c = \frac{M_y \cdot Y}{I_z} + \frac{M_z \cdot Z}{I_y}$$

POSICION DE LA LINEA NEUTRAL:

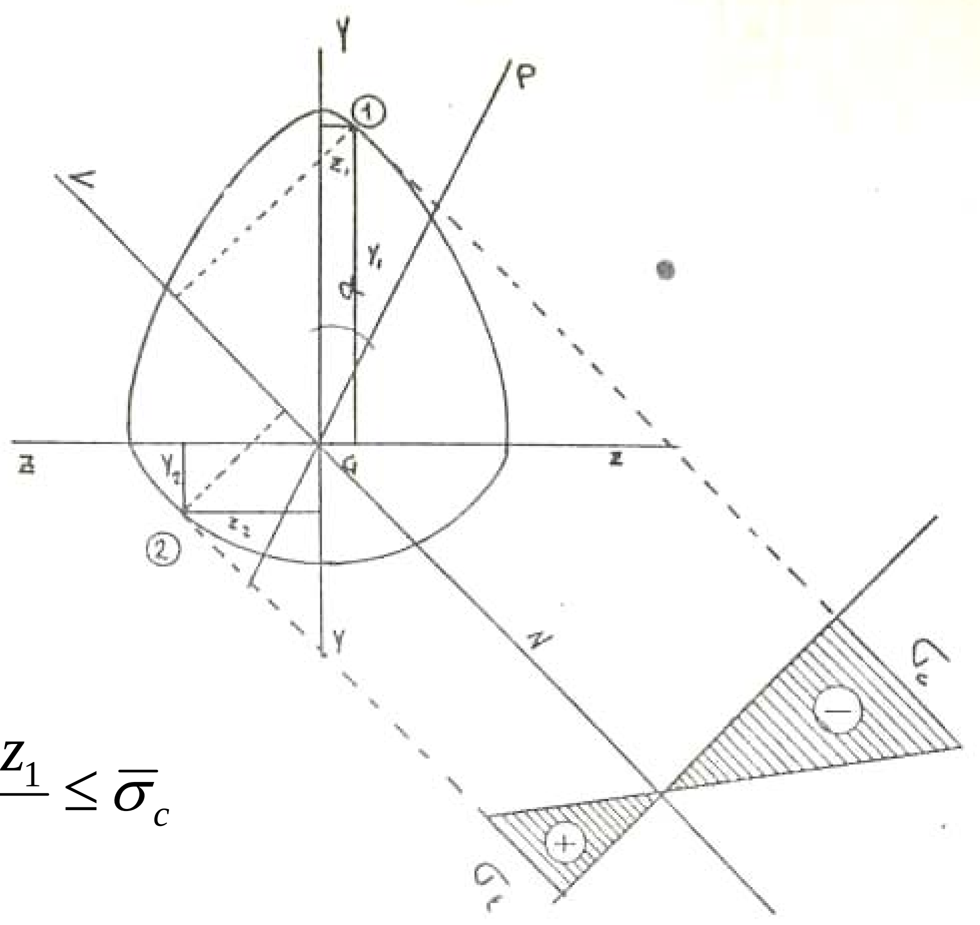
$$\frac{M \operatorname{Cos}\alpha \ Y}{I_z} + \frac{M \operatorname{Sen}\alpha \ Z}{I_y} = 0$$

$$\frac{Y}{Z} = - \frac{\operatorname{Sen}\alpha}{\operatorname{Cos}\alpha} \cdot \frac{I_z}{I_y}$$

$$Y = - \operatorname{tg}\alpha \cdot \frac{I_z}{I_y} \cdot Z$$

$$\operatorname{tg}\beta = - \operatorname{tg}\alpha \cdot \frac{I_z}{I_y}$$

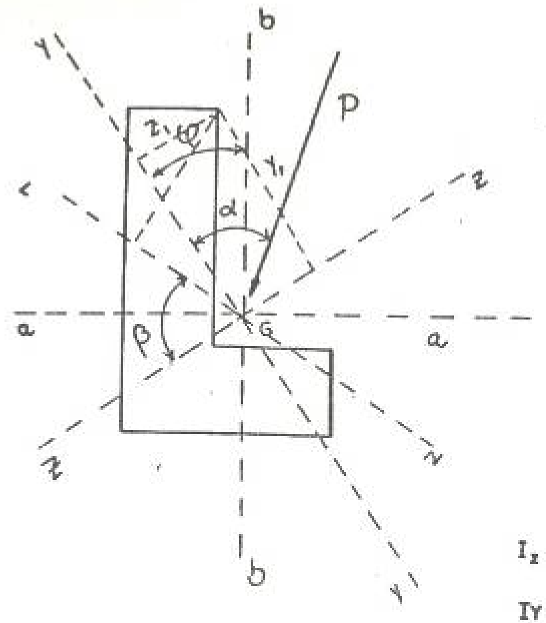
## VERIFICACION DE TENSIONES




$$\sigma_c = \frac{M_y . y_1}{I_z} + \frac{M_z . z_1}{I_y} \leq \overline{\sigma}_c$$

$$\sigma_t = \frac{M_z . y_2}{I_z} + \frac{M_y . z_2}{I_y} \leq \overline{\sigma}_t$$

# Verificación de Tensiones




① DETERM DE G.

② DETERM. DE $I_{aa}$ ; $I_{bb}$ ; $I_{ab}$

③ DETERM. EJES Y ; Z (PRINCIPALES)

$$tg\ 2\varphi = \frac{2 I_{ab}}{I_a - I_b}$$

④ DETERM. $I_z$ ; $I_y$ (PRINCIPALES)

$$I_z = I_b \cos^2\varphi + I_a \operatorname{Sen}^2\varphi - I_{ab} \operatorname{Sen} 2\varphi$$

$$I_Y = I_b \operatorname{Sen}^2\varphi + I_a \cos^2\varphi - I_{ab} \operatorname{Sen} 2\varphi$$

⑤ DETERM. $M_y$ ; $M_z$

$$M_y = M \cos\alpha$$

$$M_z = M \operatorname{Sen}\alpha$$

⑥ DETERM. L.M

$$tg\beta = \frac{I_z\, tg\alpha}{I_y}$$

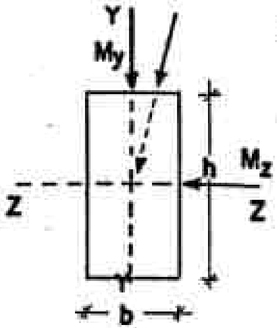
⑦ DETERMINACION PUNTOS MAS SOLICITADOS

⑧

$$\sigma_{max} = \frac{M_y Y_1}{I_z} + \frac{M_z Z_1}{I_y} \leq \bar{\sigma}$$

$$\bar{\sigma} \geq \frac{M_y}{W_z} + \frac{M_z}{W_y} \qquad \frac{W_z}{W_y} = C$$

$$\boxed{W_z = \frac{M_y + C M_z}{\bar{\sigma}}}$$




**SECCIONES RECTANGULARES**

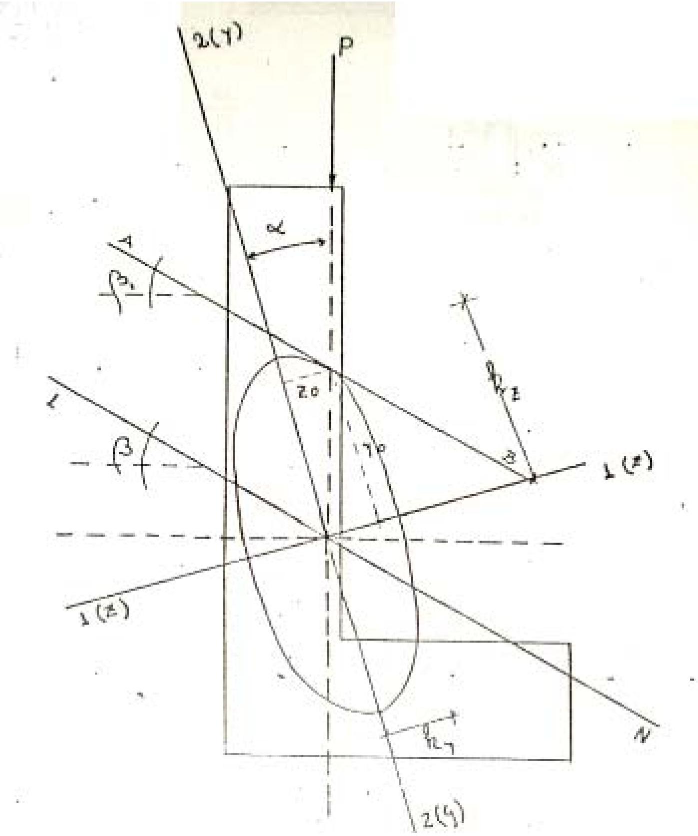
$$C = \frac{W_z}{W_y} = \frac{\frac{b\,h^2}{6}}{\frac{h\,b^2}{6}} = \frac{h}{b} = 1.2 ; \ldots\ldots ; 1.4$$

$$C = \frac{h}{b} = \frac{M_y}{M_z} \quad \text{(SECCION MINIMA)} \qquad W_z \geq \frac{2 M_y}{\bar{\sigma}}$$

**PERFILES LAMINADOS**

| | | |
|---|---|---|
| [ | 12 a 16 ...... | C = 6 |
| | 18 a 30 | C = 7 |
| I | 8 a 12 | C = 7 |
| | 13 a 24 | C = 8 |
| | 25 a 34 | C = 9 |
| | 36 a 60 | C = 10 |
| I ALAS ANCHAS | 50 a 100 – | C = 6 → 13 |

# UTILIZACIÓN DE LA ELIPSE CENTRAL DE INERCIA PARA LA DETERMINACIÓN DE LA LINEA NEUTRA




ECUACION DE LA ELIPSE CENTRAL DE INERCIA : $\frac{y^2}{k_z^2} + \frac{z^2}{k_y^2} = 1$

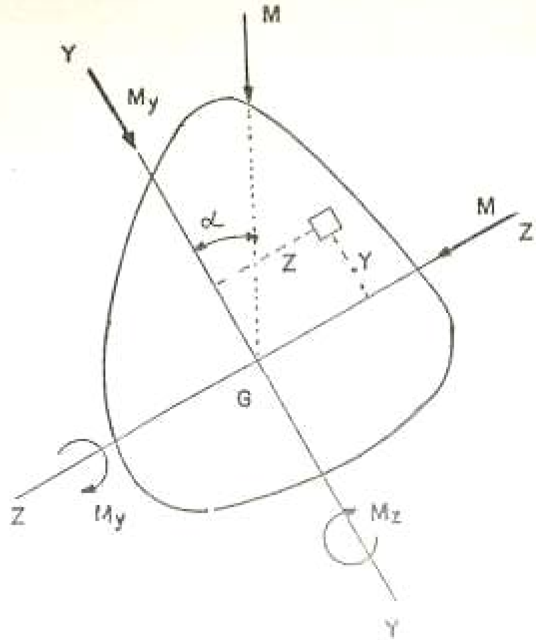
ECUACION DE A B : $\frac{Y_o\, Y}{k_z^2} + \frac{Z_o\, Z}{k_y^2} = 1$

$$tg\,\beta_1 = -\frac{B}{A} = -\frac{Z_o\, k_z^2}{k_y^2\, Y_o} = \frac{Z_o}{Y_o} \cdot \frac{I_z}{I_y} = tg\,\alpha \cdot \frac{I_z}{I_y}$$

Y COMO LA EC. DE LA L.N : $tg\,\beta = tg\,\alpha \cdot \frac{I_z}{I_y}$

$$\boxed{\beta = \beta_1}$$

# EJES BARICENTRICOS CUALQUIERA




$$\int_A \sigma \, dA = 0$$

$$\int_A \sigma \, Y \, dA = M_y$$

$$\int_A \sigma \, Z \, dA = M_z$$

$$\sigma = a + bY + cZ$$

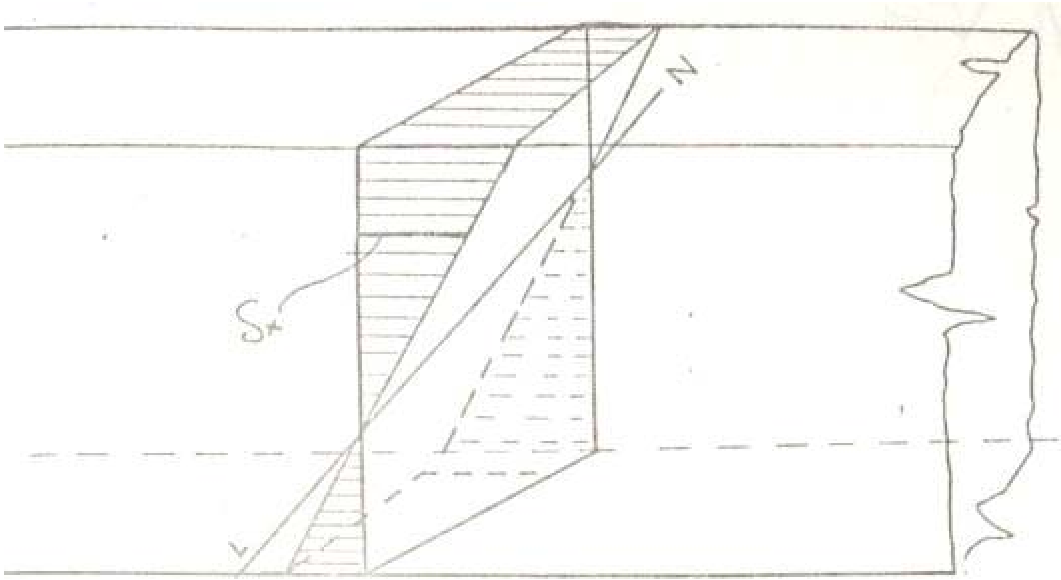
$$\int_A \sigma \, dA = 0 \;;\quad \int_A a \, dA + \int_A bY \, dA + \int_A cY \, dA = 0 \;;\quad a = 0$$

$$\int_A \sigma \, Y \, dA = M_y \;;\quad b\int_A Y^2 \, dA + c\int_A YZ \, dA = M_y \;;\quad b \cdot I_z + c\, I_{zy} = M_y$$

$$\int_A \sigma \, Z \, dA = M_z \;;\quad b\int_A YZ \, dA + c\int_A Z^2 \, dA = M_z \;;\quad b\, I_{zy} + c\, I_y = M_z$$

$$b = \frac{M_y I_y - M_z I_{zy}}{I_z I_y - I_{zy}^2} \qquad c = \frac{M_z I_z - M_y I_{zy}}{I_z I_y - I_{zy}^2}$$

$$\boxed{\sigma = \frac{M_y I_y - M_z I_{zy}}{I_z I_y - I_{zy}^2} \cdot Y + \frac{M_z I_z - M_y I_{zy}}{I_z I_y - I_{zy}^2} Z}$$

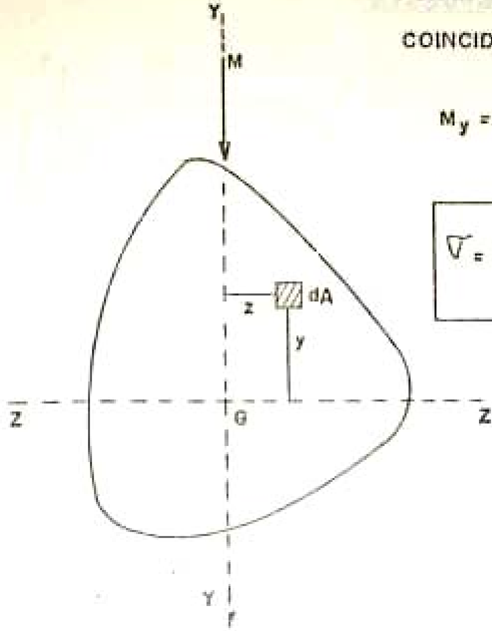



$$\sigma_x = \varepsilon \, dX_1 = (K_1 \sigma) \, dX_1$$

$$\delta_x = \varepsilon \, dX = (a_1 + b_1 y + c_1 z) \, dx$$

$$\sigma = \varepsilon E = E a_1 + E b_1 y + E c_1 z$$

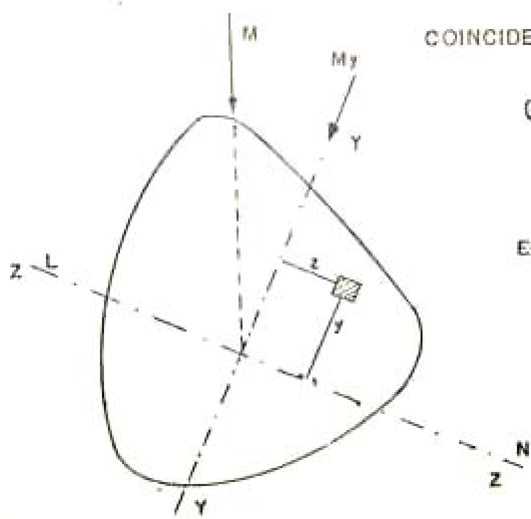
$$\underline{\sigma = a + b y + c z}$$




FORMULA REFERIDA A UN EJE COINCIDENTE CON LA CARGA

$M_y = M$ ; $M_z = 0$

$$\sigma = \frac{M\,(I_y \cdot Y - I_{zy} \cdot Z)}{I_z\, I_y - I_{zy}^2}$$




FORMULA REFERIDA A UN EJE COINCIDENTE CON LA L.N.

$\sigma = KY$ ; ENTONCES EN

$\sigma = by + cz$ ; $c = 0$

ENTONCES $M_z\, I_z - M_y\, I_{zy} = 0$

$$M_z = \frac{I_{zy}}{I_z} M_y$$

$$b = \frac{M_y\, I_y - M_z\, I_{zy}}{I_z\, I_y - I_{zy}^2} = \frac{M_y}{I_z}$$

$$\sigma = \frac{M_y \cdot Y}{I_{LN}}$$

# Próxima Clase: Flexión oblicua, línea elástica plana, línea elástica alabeada

Fin